DYNAMICS OF ROLLER BEARINGS CONSIDERING ELASTOHYDRODYNAMIC FORCES

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DYNAMICS OF ROLLER BEARINGS

CONSIDERING ELASTOHYDRODYNAMIC FORCES

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N_{Re} Reynolds number

p Pressure, psi

po Maximum Hertzian pressure, psi

P Radial force, lb_f

Q Load on a roller, lb_f

Qm See Equation 2-45

R Radius of the rollers, in.

R_p Radius of the center of the rollers. It is considered the radius of the centroidal line of radial sections of the cage.

S Ratio R/Rp

Ta Environment temperature, ^oF

Ti Inner race temperature, ^oF

To Outer Race temperature, equation 2-19, ^oF

To Entraining oil temperature, ^oR

TC Traction Coefficient

u Velocity at contact point, in/sec.

U Dimensionless velocity

v Sliding velocity at the contact, in/sec.

V Dimensionless sliding velocity

Vol Volume, in^3

W Dimensionless radial force

x Position, in.

X Pressure ratio po/110,000

Z Number of rollers in the bearing

Greeks

α	Pressure viscosity exponent, 1/psi
α_t	Lineal coefficient of thermal expansion, in/^{o}F
β	Temperature viscosity exponent 1/^{o}F
β	Angle defined in Figure 1
γ	Combined temperature-pressure viscosity exponent, 1/^{o}F. psi
δ	Displacement, in.
Δ	Effective clearance, in.
Δi	Unmounted bearing clearance
λ	Ratio sliding velocity/entraining velocity
μ	Coulomb coefficient of friction
η_o	Basic viscosity at 1 ata and 100^{o}F
$\dot{\omega}$	Angular acceleration, rad/sec^2
ρ	Density slug/in^3
σ	Poisson ratio
ϕ_s	Side leakage effect correction factor
ϕ_t	Thermal effect correction factor
Ψ	Angular position, see Figure 1

Subscripts

i	Refers to inner race, or inner race contact
o	Refers to outer race or outer race contact
j	Refers to roller j . j = 1 is the roller directly under the load
ep	Refers to epicyclic condition

CHAPTER I

INTRODUCTION

In the design, selection and performance analysis of rolling bearings it is very important to know the velocities of its elements and mainly the degree of sliding between them, because sliding is intrinsically related to friction loses, heat generation, temperature of operation and wear. The importance of this knowledge becomes more important with the every day increase in operating speeds of machines like motors, turbines, textile and other machinery where rolling bearings are used.

Many analytical and experimental investigations on this topic have been done; some of the lately published papers [1, 2, 3, 4] analyze and present solutions to this problem, valid under certain circumstances not always found in practical applications.

Poplawski [3] and Rumbarger et al. [4] presented solutions considering that the cage is stationary (or load rotating at cage speed). This assumption is consistent with Bonnes's [2] measurements of spinning speeds but it is not representative of the normal practical situation where the load is stationary while cage and rollers rotate.

These two above solutions seem to be the most complete ones known up to the present time.

Scope of This Investigation

The purpose of this work is to develop a computational model, which applying current elastohydrodynamic (EHD) knowledge about the determination of traction forces, to be able to predict the velocities of roller bearing elements when these are working under the normal conditions found in industrial applications. The author's intention is that the model be useful in design as well as in experimental works.

Tow main characteristics of the model here worked out make it quite different from previous approaches:

(a) The load and outer ring of the bearing are considered stationary while the cage and inner ring rotate. As the rollers are moving with the cage, the spinning acceleration of a roller at a given position is no longer zero. This acceleration depends on the traction forces at the contacts which in turn are determined by the radial load and the velocities. As radial forces are a function of the radial position of the roller, the acceleration will be some function of the position, but not of time if the bearing is in steady state. The spinning speed of a roller at any position is therefore the result of its acceleration at the previous points and of the time (determined by the cage speed) required to reach the position under study. From this approach it turns out that the spinning speed of the roller is a periodic function of the position.

(b) Due to the movement of cage and rollers in a fluid, there is a drag force which tends to accelerate the cage. The magnitude of the drag force depends on many factors: geometry, cage speed, temperature, fluid surrounding the moving elements, etc. It is very difficult to estimate this drag accurately. In this work a varying drag coefficient is used to determine a functional relation between cage drag and cage speed, so that known the proper drag coefficient for a given application, the operating velocities may be found, and conversely. The cage force calculated by applying the model used by Rumbarger [4] is also indicated.

The main assumptions and limitations under which this work is carried out are listed below, although some others will be stated when used in the derivations throughout the text.

1. The imbalances and geometrical imperfections are negligible.

2. Clearance between roller and cage is zero.

3. Cage is not accelerated at any moment: Nc = constant.

4. Outer ring and load are stationary or they rotate at the same speed, and the reference system is fixed on the outer ring.

5. The fluid has an exponential pressure - viscosity relation.

6. The assembly clearances between rollers and races are known.

7. The working temperatures of inner and outer rings and assembly temperatures are known. These temperatures determine the working clearance.

8. The friction between rollers and cage is of the Coulomb type.

9. The Higginson's [5] elastohydrodynamic (EHD) relations among U, V, G, W and traction forces are accepted at low Hertzian pressures.

10. Higginson's expression for the film thickness is modified as proposed by Cheng [6] to take into consideration the oil temperature rise through the contact. Side leakage is negligible.

11. The load sharing among rollers is determined by the elastic deformations, their compatibility and the static equilibrium conditions.

12. Velocities and accelerations of the axes of the rollers are

the same as the centroidal line of the cage.

13. The spinning speed of the rollers is not affected by drag forces from the surrounding gases.

CHAPTER II

FORCES IN ROLLER BEARINGS

This chapter studies three aspects of the forces in a roller bearing:

a. The load distribution of the applied load among the rollers,

b. The forces on the cage, and

c. The nature of the forces between rollers and races and the roll of the velocities at the contact points in the determination of these forces.

The Load Distribution

According to assumption 11 the part of load taken by each roller is determined considering the elastic deformation of the rollers and rings, the condition of static equilibrium and the geometrical compatibility of the deformations.

Consider Figure 1. O is the center of the outer ring of the bearing; it is considered fixed. O' is the center of the inner ring displaced a distance $\delta z1$ from O under the action of the applied load P. Circle C1 represents the undeformed outer race surface when there is no clearance in the bearing. Note that C1 also represents the outer envelope of the rollers when undeformed. C2 is the actual undeformed outer race when there is a diametrical clearance Δ. C3 is this race when the bearing is deformed under the centrifugal force of the rollers only. C4 would be the position of C1 when displaced due to the load P,

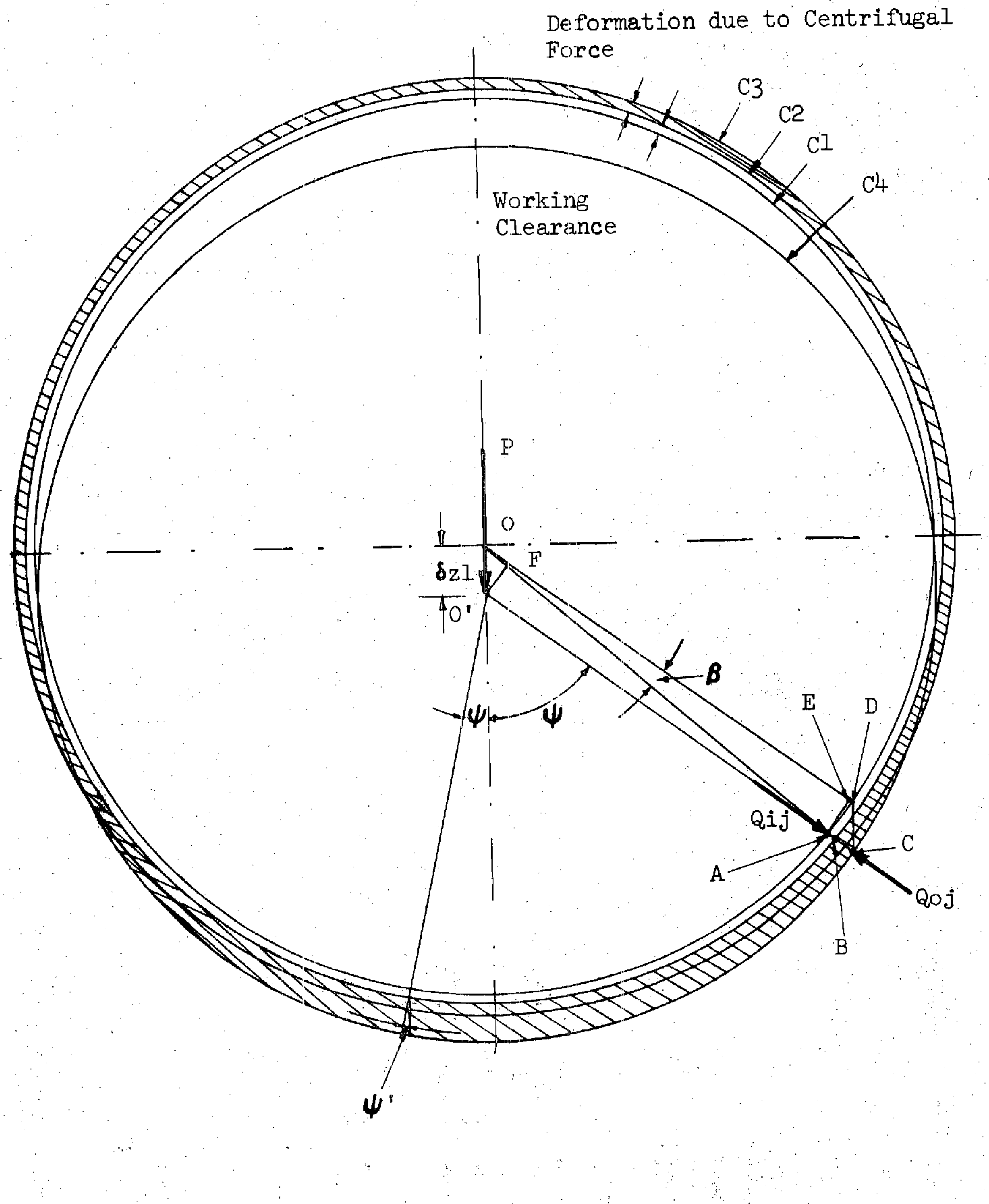

Figure 1. Deformation in a Roller Bearing

if it could displace without any deformation. The shaded area represents the total elastic deformations of the bearing, i.e., of inner ring, rollers and outer ring.

At the angular position Ψ, the elastic radial deformation is:

$$\delta r = BC - AB - AC - \Delta/2 \tag{2-1}$$

$$AC = O'C - O'A = Rp(1 + 5) - O'A \tag{2-2}$$

But

$$\text{Sin } \beta = \frac{AE}{Rp(1 + S)} = \frac{\delta zl \text{ Sin } \Psi}{Rp(1 + S)} \tag{2-3}$$

or

$$\beta = \text{arc Sin}\left[\frac{\delta zl \text{ Sin } \Psi}{Rp(1 + S)}\right] \tag{2-4}$$

Now:

$$O'A = Rp(1 + S) \text{ Cos } \beta - \delta zl \text{ Cos } \Psi \tag{2-5}$$

and

$$AC = Rp(1 + S)(1 - \text{Cos } \beta) + \delta zl \text{ Cos } \Psi \tag{2-6}$$

The total elastic deformation becomes

$$\delta r = Rp(1 + S)(1 - \text{Cos } \beta) + \delta zl \text{ Cos } \Psi - \Delta/2 \tag{2-7}$$

Usually, the first term in the right hand side of equation 2-7 does not appear in the literature on this topic. This is not relevant when the position Ψ is small but it becomes important when the value of Ψ

is near $\pi/2$ and in some cases (where Δ is a small negative quantity) it may make the difference between the convergence or not of the numerical process to calculate the load distribution.

The total radial elastic deformation is the sum of those at the inner and outer contacts:

$$\delta r = \delta i + \delta o \tag{2-8}$$

It is known [7] that the deformation in the case of the line contact in bearings may be calculated as

$$\delta = (Q/1.14 \times 10^7\ L^{8/9})^{.9} \tag{2-9}$$

Applying the foregone equation to roller "j", its deformation at the inner contact is

$$\delta ij = (Qij/1.14 \times 10^7/Le^{8/9})^{.9} \tag{2-10}$$

The deformation in the outer contact will be

$$\delta oj = \delta rj - \delta ij \tag{2-11}$$

Substituting (2-7) into (2-11)

$$\delta oj = Rp(1 + S)(1 - \mathrm{Cos}\ \beta) + \delta zl\ \mathrm{Cos}\ \Psi j - \Delta/2 - \delta ij \tag{2-12}$$

Each roller must be in radial equilibrium. Referring to figure 3-2, this equilibrium condition is

$$Qij = Qoj - Pc - \mu FcNj/|Nj| \tag{2-13}$$

Where the centrifugal force Pc is

$$Pc = \tfrac{1}{2}\rho(\pi R^2 Lt)(2\pi Nc/60)^2\ Rp \qquad (2\text{-}14)$$

Fc will be determined later.

Replacing the forces Qij and Qoj as functions of the deformation δij, equation 2-14 becomes:

$$\frac{Pc + \mu Fc Nj/|Nj|}{K} + \delta ij^{1.11} - [Rp(1 + S)(1 - \cos \beta j)$$

$$+ \delta z1 \cos \Psi j - \Delta/2 - \delta ij]^{1.11} = 0 \qquad (2\text{-}15)$$

Where

$$K = 1.14 \times 10^7\ Le^{8/g} \qquad (2\text{-}16)$$

Equation 2-15 represents a set of Z equations in Z + 1 unknowns: Z deformations δij plus the displacement δzl. To complete the set of Z + 1 equations, the condition of static equilibrium of the inner ring is used. In the present case it reduces to the fact that the sum of the vertical components of the radial forces Q must be equal to the applied load P, or

$$P = \sum_{j=1}^{Z} Qij \cos \Psi j \qquad (2\text{-}17)$$

Substituting Qij in terms of δij equation 2-17 becomes:

$$\sum_{j=1}^{Z} \delta ij \cos \Psi j - P/K = 0 \qquad (2\text{-}18)$$

The system of nonlinear transcendental equations 2-15 and 2-18 was solved numerically be a subroutine [8] which applies the Newton-Raphson method [9, 10].

In equation 2-15 the effective clearance Δ appears. It was determined assuming that the unmounted bearing clearance was known, and it was also assumed that the working temperatures at the inner and outer races were 190 and 150°F. The fits between inner ring and shaft, and between outer ring and housing were selected as the average of the standard allowed fits calculated from the standard tolerances for shafts and bearing recommended by ISO. The procedure followed to calculate Δ was that given by Harris [7], considering thermal and force fit effects. The final equation was:

$$\Delta = \Delta i - 2\ Rp\ Inh(1 + S)/Dh - Ins\ Ds/(2\ Rp(1 + S))$$

$$+ \alpha t\ [(1 + S)(To - Ta) - (1 - S)(Ti - Ta)]\ Rp \qquad (2\text{-}19)$$

Actually the inner and outer ring working temperatures must be determined via an analysis of the energy dissipation and of the thermal equilibrium in the bearing. This analysis is out of the scope of the present work.

Forces on the Cage

Two kinds of forces act on the cage: the forces from the rollers and drag forces due to the movement of cage and rollers in a fluid.

Forces Between Cage and Rollers

As the roller speed changes along a revolution, so do the forces between roller and cage. This force may change its sense too. The numerical value of this force may be found only when the equilibrium state of the bearing is known. In this sense this is one of the items looked for in this investigation.

The force between rollers and cage causes friction between them. This friction is assumed to be of the Coulomb type (assumption 8) and it is also assumed that the sum of all those friction forces (radial) on the cage is zero. Actually the friction forces may cause the cage to rotate excentrically and/or to oscilate radially, nevertheless as excentricity or oscilation of the cage have not been reported. It is believed that the assumption does not have significant influence in the prediction of the dynamical operational behavior of the cage.

<u>Drag Force</u>

This force results from the motion of the cage in its surrounding fluid (a mixture of lubricant and air, in most of the cases). The drag is difficult to estimate mainly because of the unknown proportions and properties of the fluid and the complex geometry of the cage-rollers assembly. Two approaches may be followed to calculate the drag forces:

a. Using a drag coefficient, to be determined for each circumstance, the drag force is:

$$\text{Drag} = (\tfrac{1}{2})\ Cd\ \rho_f U^2 A \qquad (2\text{-}20)$$

The area may be taken as:

$$A = 4\pi\ Rp^2\ S \qquad (2\text{-}21)$$

and the velocity is:

$$u = 2\pi\ Rp\ Nc/60 \qquad (2\text{-}22)$$

b. A second approach may be to apply to model used by Rumbarger et al. [4]. Two simplifications may be introduced in the model: the density of the mixture is the average of the density in the land region and that in the other parts of the bearing. The second simplification is that the drag on each roller due to the spinning speed is not important, compared to the other forces acting on the roller, for the determination of the spinning speed of the rollers.

The force on the cage is:

$$\text{Drag} = \tfrac{1}{2}\,\rho_{\text{aveg.}} \cdot f\,U^2 A \tag{2-23}$$

$$\rho_{\text{aveg}} = \frac{\rho\text{oil} \cdot \text{Dec}(1 + \text{Dec})}{0.8 + 1.2\ \text{Dec}} \tag{2-24}$$

Dec is the ratio volume of oil to total volume in the mixture.

$$f = 16/N_{Re} \qquad N_{Re} < 2500 \tag{2-25}$$

$$f = 3[N_{Re}/2500]^{0.85596} \times 16/N_{Re} \qquad N_{Re} \geq 2500 \tag{2-26}$$

$$N_{Re} = \frac{2\pi Nc}{60} \cdot Cc \tag{2-27}$$

Cc being the clearance between the equivalent cage-rollers cylinder and the races.

<u>Equilibrium of the Cage</u>

For the steady state operation of the bearing it may be assumed that the cage is not accelerated, i.e., $dNc/dt = 0$. On the average this is true and it is also true at least Z times in each revolution. For a nonsymmetrical position of the rollers in respect to the load, the load distribution is not symmetrical. The traction forces distribution is also not symmetrical. This fact may cause some accelerations on the cage. In any case, these accelerations (if any) must be periodical, the period being at most $1/(ZNc)$. Further theoretical and experimental investigations may consider and eventually elucidate this point.

On this basis the cage equilibrium requires that:

$$\sum_{j=1}^{Z} Fcjrollers + Drag = 0 \qquad (2\text{-}28)$$

Traction Forces

The selection of the proper model to estimate the traction forces at the contacts presented some difficulties due to the lack of a theory or correlation simple enough to be handled without the necessity of introducing too complex and time consuming programs and with a full range applicability [12].

Some of the published theories and correlations give unrealistic results for some ranges of load [5, 13, 14]. Another difficulty was the scattering of the published data or the limited applicability to very restricted situations of other data and correlations [15, 16, 17, 18, 19, 20, 21, 22, 2]. Perhaps this situation arose because, up to the present time most of the work done on EHD has been devoted to the measurement and prediction of the fluid film thickness in the contacts, while the traction forces have not received the same attention.

In the course of the investigation two models were used to calculate the forces between rollers and races: the first one used current

EHD theories. The second model was the result of a correlation made by the author, of the data published by Trachman and Cheng [11] for the fluid XRM - 109. This fluid has been used in the course of the investigation. It was selected because their properties and traction characteristics were known.

The use of dimensionless variables have been done using the groups proposed by Higginson and Dawson [5], every contact being reduced to its equivalent between a roller and a plane as shown in Figure 2.

Elastohydrodynamic Relations

To reduce the real contact Figure 2a to the equivalent plane surface-roller contact Figure 2b, the following relations must be used:

$$R = (R_1^{-1} + R_2^{-1})^{-1} \tag{2-29}$$

$$E = 2\left[\frac{1 - \sigma_1^2}{E_1} + \frac{1 - \sigma_2^2}{E_2}\right]^{-1} \tag{2-30}$$

Four dimensionless variables were used:

Film thickness

$$H = h/R \tag{2-31}$$

Load

$$W = Q/(ELeR) \tag{2-32}$$

Velocity

$$U = \eta_o u/(ER) \tag{2-33}$$

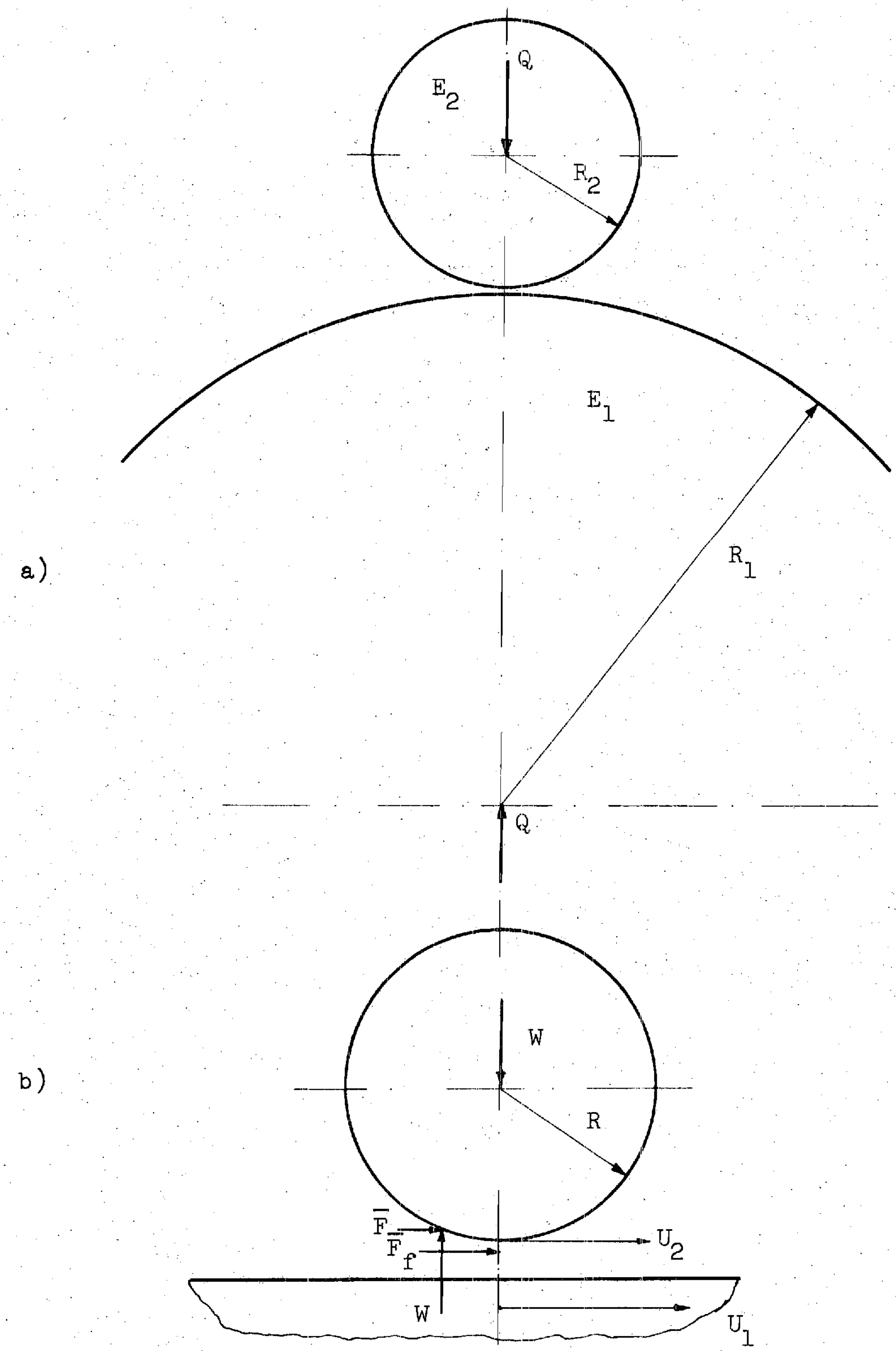

Figure 2. Equivalent Contact Geometry

$$V = \eta_o v/(ER) \tag{2-34}$$

and

$$G = \alpha \cdot E \tag{2-34}$$

The relations among these variables were those proposed by Higginson and Dawson [5], only the expression for H was modified to take into account the side leakage and thermal effects as suggested by Cheng [6]. Referring to Figure 2b:

$$F = 1.43\ U^{0.71} \tag{2-35}$$

$$F_f = -F/2 + VI/H \tag{2-36}$$

$$H = 1.6\ U^{0.7}\ W^{-a}\ F^{0.6}\ \phi_s\ \phi_t \tag{2-37}$$

"I" is the dimensionless product $(\eta/\eta_o)dA$ integrated over the contact area. To carry out this integration, it is assumed that the lubricant pressure viscosity relation is:

$$\eta = \eta_o e^{\alpha p} \tag{2-38}$$

The pressure used is that given by a Hertzian distribution. In dimensionless form the integral becomes:

$$I = \int_{-b/R}^{b/R} \exp[G\bar{p}_o(1 - (R\,\bar{x}/b)^2)^{\frac{1}{2}}]d\bar{x} \tag{2-39}$$

$\bar{x}$ is the dimensionless variable of integration

$$\bar{x} = x/R \tag{2-40}$$

$$\bar{p}_o = (W/2\pi)^{\frac{1}{2}} = p_o/E \tag{2-41}$$

$$b = 4\bar{p}_o R \tag{2-42}$$

The Film Thickness Correction Factors

Many investigators have suggested that the Higginson's expression for the film thickness must be corrected in order that it can agree with experimental data. Three factors must be taken into account to make these corrections: side leakage of the lubricant, thermal effects and high pressures. In the case under study the side leakage is not important (the Le/b relation is very large) so ϕ_s is taken as one. The pressure effect is considered in the next part of this chapter. The thermal effect is taken into account as proposed by Cheng [6]; an empirical relation was derived for oil XRM - 109 [23] which correlates very well with Cheng's factors (see Appendix C). Although other studies were considered [24, 18, 21] Cheng's factors turned out to be the most convenient in this case.

For XRM-109 the thermal correction factor is

$$\phi_t = \mathrm{Exp}[-(0.5677 + 0.0348X)Q_m^{\,(.4003 + 0.0311X)}] \tag{2-43}$$

$$X = p_o/110000 \quad \text{dimensionless} \tag{2-44}$$

$$Qm = \eta_o(u_1 + u_2)^2/(2K_f\,To) \quad \text{dimensionless} \tag{2-45}$$

Traction Forces at High Pressures

The Higginson's formula for the traction forces predicts high traction forces unrealistically when the pressures are not small. This effect has been studied by many authors [18, 26, 19, 12], but it does not seem to exist an agreement as to how to improve the prediction of traction forces when the pressures are high. In the present investigation an experimental work done by Trachman and Cheng [11] was correlated to derive an empirical equation to estimate the traction forces. After a review of the literature some works were considered to be used [20, 16, 22, 13] and finally the Trachman set of data was selected because:

1. One of the fluids used was XRM-109.

2. The traction coefficients were found under combinations of both rolling and sliding velocities of the same range of values found in the present work.

3. The range of pressures was wide enough to include most of the cases found in the current investigation.

4. They are for line contact.

The final form of the traction coefficient is (see Appendix D)

$$TC = C\lambda + D(1 - e^{-A\lambda}) \qquad (2\text{-}46)$$

being:

$$\lambda = v/Ua \qquad (2\text{-}47)$$

$$C = .0143X + .0357 \qquad (2\text{-}48)$$

$$D = 0.046 - 0.031\ e^{-0.31136X^{3.41435}} \tag{2-49}$$

$$A = 147{,}507 \text{ arc tg}(4.76979(X - 1.0597)) \tag{2-50}$$

$$+ 0.1\ (X - 1.0597) + 210$$

This formula correlates very well to the data and is perfectly valid for the range of velocities ($\lambda \leq .14$) which covers most of the cases studied. To extend the range of validity of the formula an experentially decaying expression was used going through the value of Tc at $\lambda = 0.14$ and decaying asymptotically to zero.

CHAPTER III

DYNAMICS OF THE ROLLER BEARING

The goal of this work is to determine the velocities of all elements in the bearing, general expressions for the velocities of the points of contact and the rotating speeds of every part are necessary to be derived. By applying D'Alambert principle equations for dynamic equilibrium and the accelerations are found.

The velocities, rotating speeds and accelerations are absolute (assumption # 5) and their positive senses are those indicated in Figure 3.

Kinematics of the Roller Bearing

The expressions for some of the velocities found herein are somewhat different from those found in some references [7, 4, 1]. This is due to assumption # 4.

As a point of reference, the epicyclic speeds are found from the general expressions by simply setting the slip equal to zero in the inner and outer contact.

Referring to Figure 3, the velocities at the contact are:

$$U_{1i} = \frac{2\pi NRp(1 - S)}{60} = \frac{\pi Rp}{30} \cdot N(1 - S) \tag{3-1}$$

$$u_c = \frac{2\pi RpNc}{60} = \frac{\pi Rp}{30} \cdot Nc \tag{3-2}$$

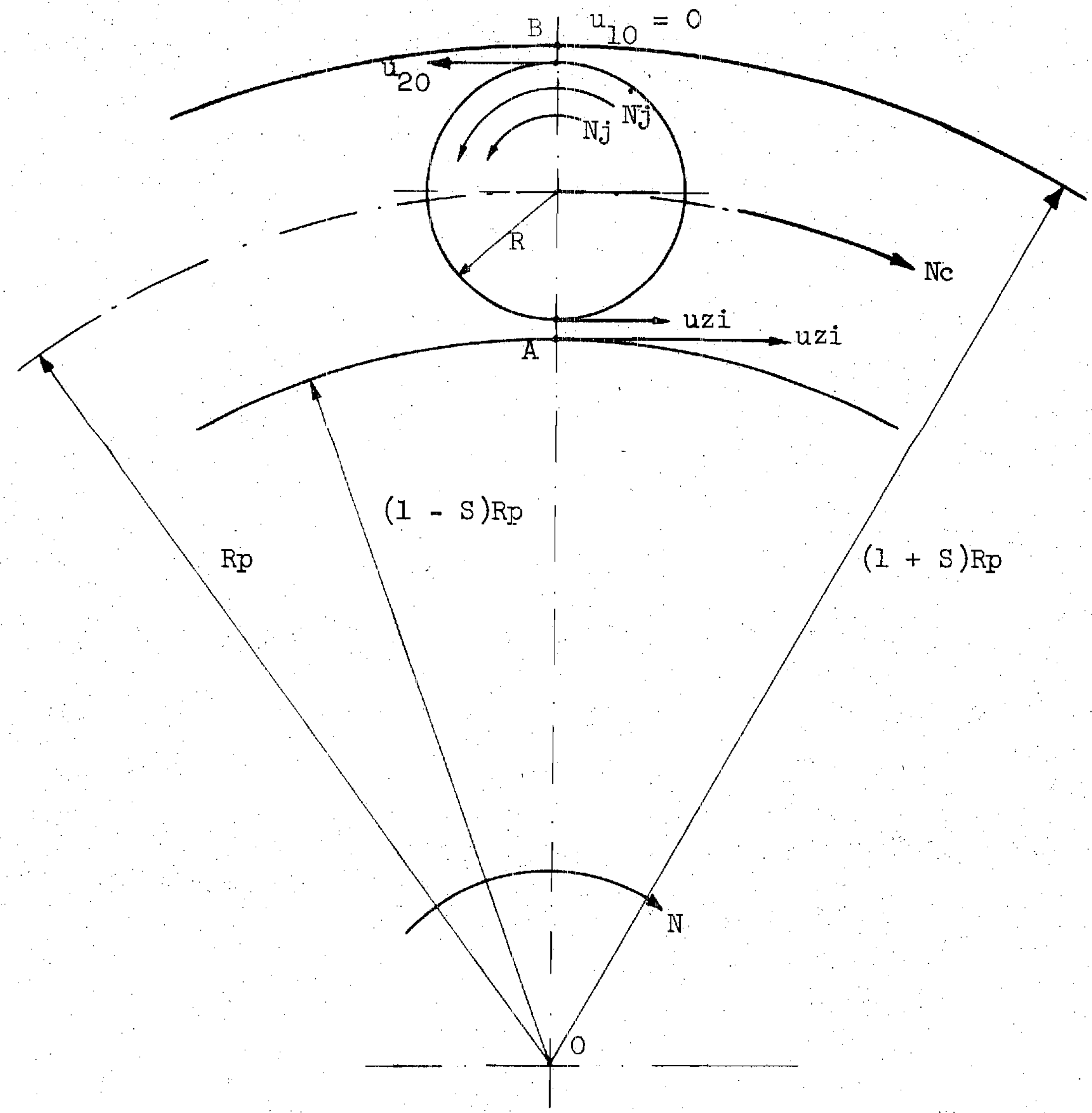

Figure 3. Velocities of Roller Bearing Elements

$$u_{2i} = \frac{\pi Rp}{30} (Nc + S \cdot Nj) \tag{3-3}$$

$$u_{2i} = u_{A2} = u_p + u_{2A/o} \tag{3-4}$$

$$u_{20} = \frac{\pi Rp}{30} (S \cdot Nj - Nc) \tag{3-5}$$

The mean rolling velocities are:

$$u_i = \tfrac{1}{2}(u_{2i} + u_{1i}) = \frac{\pi Rp}{30} [N(1 - S) + Nc + S \cdot Nj]/2 \tag{3-6}$$

$$u_o = \tfrac{1}{2}(u_{20} + u_{1o}) = \frac{\pi Rp}{30} (S \cdot Nj - Nc)/2 \tag{3-7}$$

The sliding velocities become

$$v_i = u_{2i} - u_{2i} = \frac{\pi Rp}{30} [N(1 - S) - Nc - Nj \cdot S] \tag{3-8}$$

$$v_o = \frac{\pi Rp}{30} (Nc - S \cdot Nj) \tag{3-9}$$

If there is no slip at any contact, the epicyclic speeds are:

$$Ncep = N(1 - S)/(2S) \tag{3-10}$$

$$Njep = N(1 - S)/2 \tag{3-11}$$

Eliminating N between equations (3-10) and (3-11):

$$Njep = Ncep/S \tag{3-12}$$

Dynamics of the Rollers

Figure 4 shows the forces acting on a roller. The condition of dynamic equilibrium requires that $\Sigma F_r = 0$. as it is assumed that there

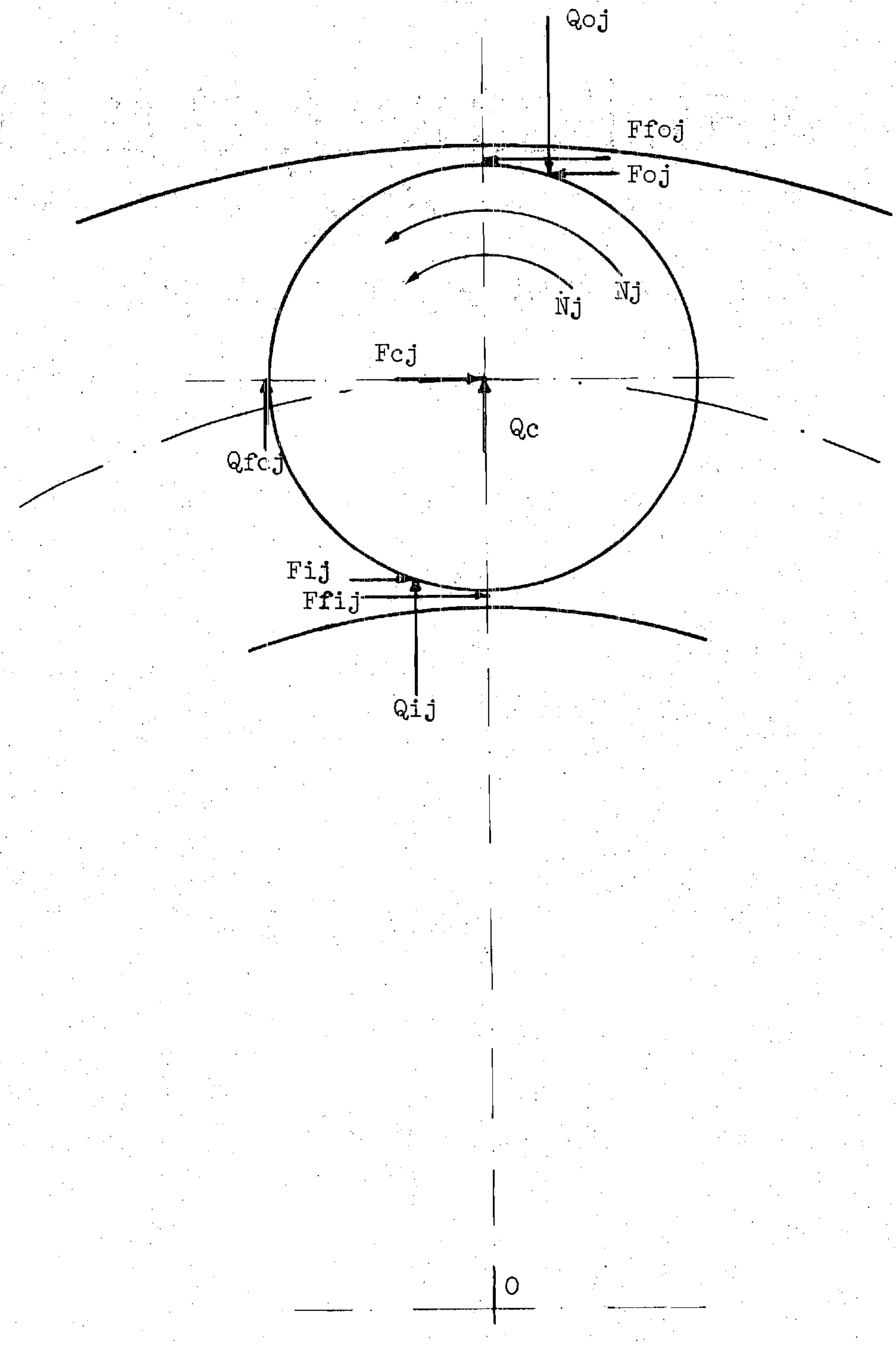

Figure 4. Forces on a Roller

is not any clearance between roller and cage and according to the assumption made that the cage is not accelerated at any time, the summation of forces in tangential direction must be zero $\Sigma F_t = 0$. The only acceleration suffered by the roller is the spinning acceleration. Writing down the previous statements as equations:

$$Qij + Qc + Q_{fcj} - Qoj = 0 \tag{3-13}$$

$$Fij + F_{fij} - Foj - F_{foj} + Fcj = 0 \tag{3-14}$$

$$S \cdot Rp(Fij + F_{fij} + Foj + F_{foj} - \mu|Nj||Fc|/Nj) - J\omega j = 0 \tag{3-15}$$

Note that

$$Qfcj = \mu\ Fc\ |Nj|/Nj \tag{3-16}$$

Adopting the assumption [7, 5, 3, 4] that the forces Fij and Qij have a resultant going through the center of the roller (idem for Qoj and Foj), equation 3-15 reduces to

$$J\omega j = S \cdot Rp\ (F_{fij} + Fo_{fj} - \mu\ |Nj||Fc|/Nj \tag{3-17}$$

The last assumption is true when the roller is rigid, but if the surface is feformed the direction of this resultant may change its direction. Furthermore, depending on the magnitudes of the velocities (dimensionless) and pressures, the center of pressure and the pick of pressure may be displaced and it is possible that the force will be shifted an unknown distance. At the present time it is not possible to prove the existence of this shifting nor magnitude. On this basis the assumption is accepted.

Radial Direction

The centrifugal forces Qc is equal on all rollers and is

$$Qc - m \; Rp \; \omega c^2 \tag{3-18}$$

In dimensionless form

$$Wc = \frac{\rho \; Vol \; Nc^2 \; Rp}{Le \; E \; Ri} \left(\frac{2\pi}{60} \right)^2 \tag{3-19}$$

For steel rollers

$$Wc = 0.255 \; S^2 \; L_t \; Rp^3 \; Nc^2 \times 10^4/(Le \; E \; Ri) \tag{3-20}$$

$$Wc = C_1 \; Nc^2 \tag{3-21}$$

The dimensionless friction force is

$$Wfcj = \mu \; |Nj| \; \overline{F}cj/Nj \tag{3-22}$$

Finally

$$Wij + C_1 \; Nc^2 - Ro \; Woj/Ri + Wfcj = 0 \tag{3-23}$$

The radial force in the outer contact is hence

$$Woj = Ri \; (Wij + C_1 \; Nc^2 + \mu |Nj| \; \overline{F}cj/Nj)/Ro \tag{3-24}$$

Tangential Direction

If the cage force on the roller, Fcj from equation 3-14 is non-dimensionalized with respect to the inner contact equivalent radius Ri, it is

$$\overline{F}cj = \frac{Fc}{Le\ E\ Ri} Ro\ (\overline{F}oj + \overline{F}foj)/Ri - \overline{F}ij - F_{fij} \tag{3-25}$$

If the Higginson's formulas are used to relate forces to velocities, then

$$\overline{F}c = Ro(.715\ U^{0.71} + Voj\ Ioj/Hoj)/Ri \tag{3-26}$$

$$- .715\ U^{.71} - Vij\ Iij/Hij$$

being

$$Uij = \frac{\eta_o\ uij}{Le\ E\ Ri} \tag{3-27}$$

$$Voj = \frac{\eta_o\ voj}{Le\ E\ Ro} \tag{3-28}$$

$$Hij = 1.6\ Uij^{.7}\ G^{.6}\ W^{-a}\ \phi_{tij} \tag{3-29}$$

Expressions for the remaining variables in 3-26 are similar to the last ones but substitute the proper sub-indexes.

If the traction coefficient approach is used, the dimensionless cage force on the roller is

$$\overline{F}cj = RoTCoj\ Woj/Ri - TCij\ Wij \tag{3-30}$$

Sometimes it is necessary to know the derivative of the cage force in respect to the inner ring speed N: it is

$$\frac{\partial \overline{F}cj}{\partial N} = Wij\ \frac{\partial TCij}{\partial N} \tag{3-31}$$

as $\partial TCoj/\partial N$ is zero.

Rollers Acceleration

From equation 3-17 the acceleration of the spinning speeds of the rollers as a function of the dimensionless forces is, in rad/sec^2

$$\dot{\omega}j = S\ Rp\ Le\ RiE(Ro\ \overline{F}_{foj}/Ri + \overline{F}_{fij} - \mu\ |Nj|\,|\overline{F}cj|/Nj) \qquad (3\text{-}32)$$

When the Higginson's formulas are used in 3-32, it becomes

$$\dot{\omega}j = S \cdot Rp\ Le\ E\ Ri[Ro(.715\ Uoj^{.71} + Voj\ Ioj/Hoj)/Ri \qquad (3\text{-}33)$$

$$+ .715\ Uij^{.71} + Vij\ Iij/Hij - \mu|Nj|\,|\overline{F}cj|/Nj]$$

On the other hand, when traction coefficients are used the acceleration is

$$\dot{\omega}j = S\ Rp\ Le\ E\ Ri(TCoj\ Ro\ Woj/Ri + TCij\ Wij - \mu|Nj|\,|\overline{F}jc|/Nj) \qquad (3\text{-}34)$$

CHAPTER IV

METHOD OF ATTACK

The Spinning Speed Function

Finding the spinning speeds and the inner ring speed implies a method of successive approximations because the forces and accelerations are functions of the speeds looked for. In the present study the spinning speed acceleration for a given roller (at a given position) is not equal to zero but it is constant at each position if the bearing is in a steady state, as it is assumed.

The change in acceleration from one position to another must be such that the new speed and acceleration in the new position be the result of the previous values. Furthermore, this change must be continuous and periodical with a period equal to that of the cage.

Known the roller spinning acceleration as a function of time, it is easy to put it as a function of position, so that when integrated it gives the spinning speed at each position of the roller. It must be remembered also that the load on each roller depends, in general, on the position of the roller measured by the angle Ψ.

The necessary transformations are:

$$\dot{\omega}j = \frac{d\omega j}{dt} = \frac{d\omega j}{d\Psi} \cdot \frac{d\Psi}{dt} \qquad (4\text{-}1)$$

But $d\Psi/dt$ is the velocity at which the roller changes its position, i.e., the cage speed. The change in the spinning speed due to the change in position is then

$$\frac{d\dot{\omega}j}{d\Psi} = 60\ \ddot{\omega}j/Nc \qquad \frac{rad}{sec}/rad \tag{4-2}$$

In revolutions per minute it is

$$\frac{dNj}{d\Psi} = 1800\ \ddot{\omega}j/(\pi Nc) \qquad \frac{rev}{min}\ /rev \tag{4-3}$$

$\ddot{\omega}j$ is given by equations 3-33 or 3.34.

If $dNj/d\Psi$ is integrated over one revolution the result must be zero as Nj is a periodic function of Ψ. The problem to solve is then reduced to find the function $Nj(\Psi)$ and the inner ring speed N, such that

$$\int_0^1 \frac{dNj}{d\Psi}\, d\Psi = 0 \tag{4-5}$$

and

$$\text{Drag} + \sum_{j=1}^{Z} Fcj = 0 \tag{4-6}$$

The Method of Solution

An analytical solution for this problem or even for any of its main parts is not possible. Thus a computerized solution was implemented by writing programs to solve the different parts of the problem and then they were integrated to bring the complete solution.

Load Distribution

The set of z + 1 nonlinear equations 2-15 and 2-17 were solved by Newton-Raphson method [8] given a set of cage forces Fij and a set estimated deformations. The criteria for convergence was that the change in each deformation from iteration to iteration had to be less than 0.1 percent. The cage forces remain constant through the process.

The found deformation was used to calculate the forces on each roller

$$Qij = 1.14 \times 10^7 \; Le^{8/9} \; \delta ij^{1.11} \qquad (4\text{-}7)$$

Solution for Speeds

Known the loads acting on the rollers, the maximum Hertzian pressures may be calculated. If at least one of the rollers were under pressures higher than 60,000 psi the acceleration was calculated using the traction coefficient correlation, otherwise the Higginson's formulas were used.

Three iterative loops, one inside the other were necessary to find the spinning speed distribution and the inner ring speed. This process began with a set of estimated inner ring and rollers speeds.

First Loop

Keeping the value of Nj_1 at $\Psi = \Psi_1 = 0$ (this position corresponds to the roller directly under the load) constant this loop calculates the forces and accelerations necessary to integrate $dNj/d\Psi$ from Ψ_1 to $\Psi_1 + \Delta\Psi$, predicting in this way the new value of Nj at $\Psi_1 + \Delta\Psi$. This new value substitutes the previous one for that position. $dNj/d\Psi$ is taken from equation 4-3 and in integrated using a sixth order formula [10].

The integration continues, step by step through a complete revolution. Keeping the first value constant, this integrating routine is repeated until the change in the last value of Nj between two successive integrations is less than .01 percent.

Second Loop

When the first loop converges, the value of Nj_1 at $\Psi = 0$ is changed using a successive replacement method [9, 27, 10] and loop one repeated. These changes in Nj_1 are made until condition 4-5 is satisfied within a .01 percent of the value of Nj_1. At the end of this loop the speed distribution and the net cage force are printed, if desired.

Third Loop

The spinning speed distribution found in the second loop is not the correct one unless condition 4-6 is satisfied. If that requirement is not satisfied the value of the inner ring speed N is changed, using a decelerated Newton-Raphson method [9], and loop two repeated again. In this step the derivatives are evaluated numerically and the criteria for convergence is that the change in the inner ring speed must be less than .01 percent.

The loop with the fastest convergency is the loop two, which usually needs only three or four iterations, while loop one has the slowest convergency requiring ten or twelve iterations in some cases.

General Procedure

The general procedure is as follows: Given an applied load P and a cage speed Nc, the load distribution is found, neglecting the friction forces between rollers and cage, but considering P and the centrifugal loads. With the found loads on the rollers kept constant the equilibrium speeds are found as described under "solution for speeds."

The friction forces of the cage on the rollers are small compared to those due to the applied load and centrifugal effect so their influence on the load distribution is very small. Nevertheless, the equilibrium speeds may be taken to find the individual cage forces and the friction forces. Taken into account these forces, a new load distribution and new equilibrium speeds are found, and so far. This general iteration usually converges after one or two iterations.

CHAPTER V

ANALYSIS OF RESULTS

Results

Appendix A has the computer programs written in Fortran V by the author: two main programs and nine subroutines. Four more subroutines from reference [8] were used but are not included in Appendix A.

The first main program is matched with the first of the two subroutines SYS2 to find the cage force for given values of load and epicyclic (no slip condition) cage speed, as function of cage slip. Slip is defined as:

$$\text{slip} = (Nc - Ncep)/Ncep \tag{5-1}$$

Noting that the variations in centrifugal force are small for small variations in cage speed and that the operating range of slip is small the results of the first main program may be interpreted as cage force as the function of N for a given Ncep, or as cage force being a function of Nc for the corresponding epicyclic shaft speed. Tables 2 to 10 are results of this program.

The second main program when used with the first of the subroutines SYS2 gives the spinning speed distribution, epicyclic speeds, total and individual cage forces, traction coefficients and slip as well as the elastic deflections, loads and film thicknesses.

Tables 11 to 13 are results obtained with this program.

When the second main program is matched with the second of the subroutines SYS2, the program gives the same type of values as in the previous case, but here it uses the drag model given in reference [4], the specified load and cage speed and an assumed inner ring speed. The tabulated results correspond to this operating point.

Table 1. Bearing Characteristics

Bore Diameter	2.5590	in.
Outer Diameter	5.5118	in.
Width	1.2992	in.
No. of Rollers	12	
Roller Diameter	.8268	in.
Roller Length	.6962	in.
Pitch Diameter	4.035	in.
Clearance	0.0	in.

These programs were run for a bearing whose characteristics are given in Table 1, lubricated with oil XRM-109 [23]. A combination of seven loads and nine speeds were the inputs. The results are given in Tables 2 to 13 and are plotted in Figures 5 to 29.

Figures 5 to 7 give the driving cage force (from the rollers) as a function of slip (or operating cage speed) for the three different shaft speeds, it also gives the drag force on the cage as a function of Nc. In each graph the parameters are the loads and the drag coefficients.

Figures 8 to 17 are plots of cage forces versus the slip (or inner ring speed) for seven cage speeds. The parameter is the applied load.

Drag force as a function of the drag coefficient is also shown.

The driving cage force presents a maximum for some value of slip, as can be seen in Figures 5 to 17, Figure 18 represents the slip at which the maximum driving cage force would be obtained versus epicyclic shaft speeds for different loads.

Figure 19 is the qualitative representation of the behavior of the bearing as the shaft velocity increases.

The deflection at the inner contact is plotted in Figures 20 to 22. Position 1 corresponds to the roller directly under the applied load.

Figures 23 to 25 give the dimensionless film thickness and finally Figures 26 - 29 give the spinning speed of the rollers as a function of the position for different loads.

Conclusions

The main conclusion drawn from this work is that it is possible to predict the speeds and forces of rolling bearing elements as well as any other desired operating parameter under practically any condition of loads and speeds. Some other particular conclusions are listed below.

Cage Forces

1. The total net force from the rollers on the cage increases rapidly as the slip increases until it reaches a maximum value. After this point, the cage force decreases very slowly as the slip is increased.
2. The existence of this maximum in the cage force - slip curve suggests that for an operating cage speed (constant drag) there may be two equilibrium shaft speeds. It is believed that in the practical range of loads

and speeds only the low slip equilibrium position is reached. 3. The slip at which the maximum cage force is reached is affected by the applied load and the epicyclic speed. An increase in load or speed brings a decrease in the value of that slip, although the effect of changes in load is much higher than that of the changes in speed. 4. There is not an adequate analytical method for determining the drag force on the cage.

Operating Speeds

5. In the present study the drag was calculated for different values of the drag coefficient so that if the drag coefficient is known (or determined) the operating speeds may be found from the charts of Figures 5 to 17 and 26 to 29. 6. A plot of the forces on the cage against cage speed such as that qualitatively suggested in Figure 19 would indicate that for a given applied load there is a maximum cage speed given by the intersection of the drag force curve and the curve going through the points of maximum cage driving force. If the shaft is rotated at a speed higher than corresponding to the maximum Nc, the cage speed decreases. This conclusion agrees with the predicted double value of slip for given cage speed and load. 7. Using Figures 26 to 29, it is possible to determine the spinning speed at each position of the rollers. 8. The minimum spinning speed is just before the roller enters the inner race contact zone, from this point it is accelerated up to a maximum, somewhere after the point of maximum load. This conclusion is the opposite of Poplawski's conclusion [3], but it agrees with Bonnes' data [2].

Influence of Centrifugal Forces

9. The increase of centrifugal forces has an effect on the load distribution and elastic deflections at the inner race similar to an increase in clearance. 10. An increase in centrifugal force increases the deformation of the roller under the load and decreases that of the rollers at the end of the loaded zone. 11. The film thickness increases when the centrifugal forces increase. 12. When a bearing is operating at the low slip point of equilibrium, an increase in centrifugal force decreases the operating cage slip.

Recommendations

In order to improve the understanding of the dynamics of the roller bearings and the ability to predict their behavior, the author encourages:

1. Experimental determination of the velocities of the elements of roller bearings in the practical range of loads and speeds, in order to have reliable data to compare with the theoretical results.

2. A theoretical and experimental investigation to determine the resistant forces on the cage and rolling elements assembly.

3. An extension of the present model to other lubricants with different traction coefficient characteristics.

4. Generalize the study to the ball bearing case.

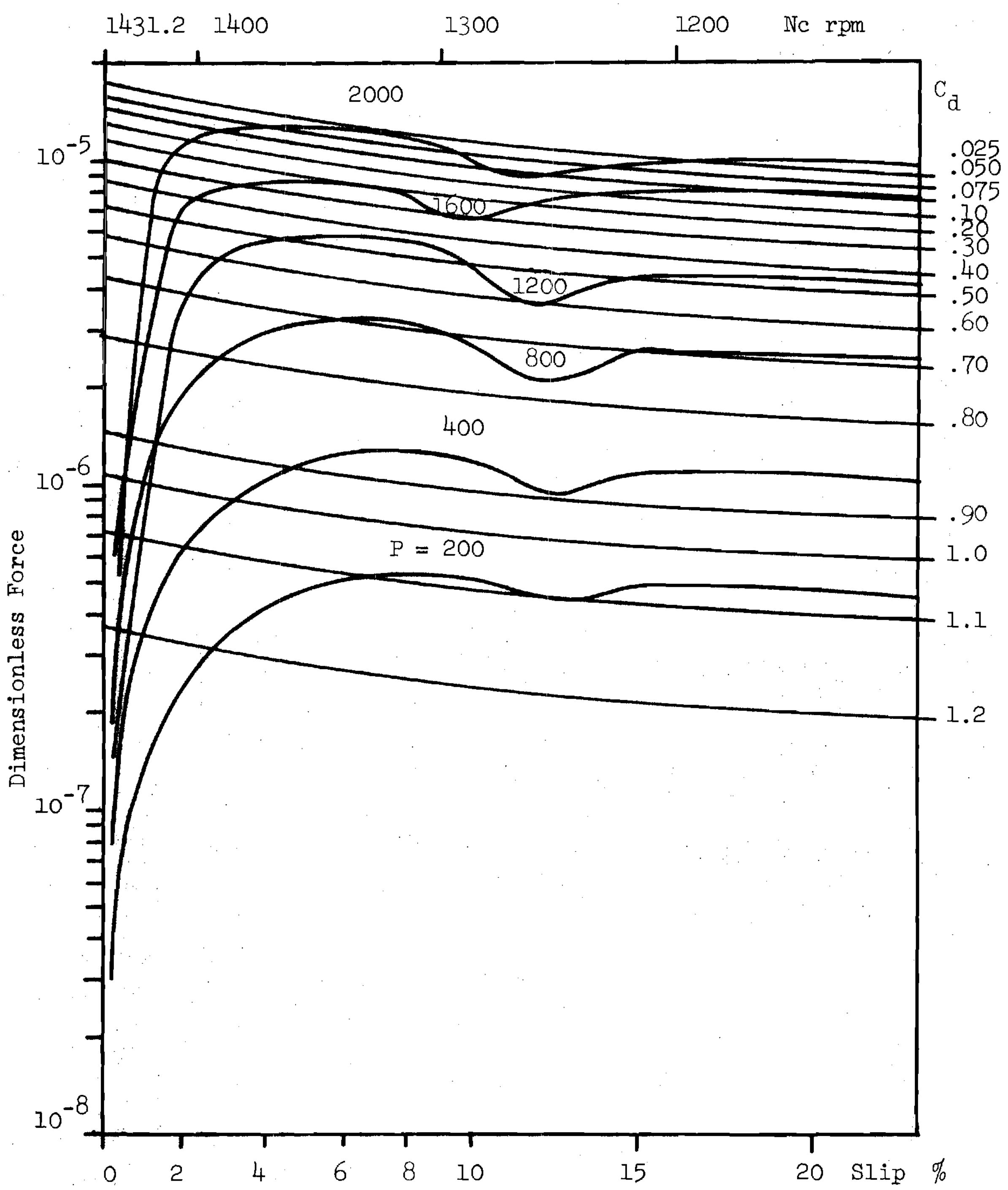

Figure 5. Cage and Drag Forces. Shaft Speed 3600 rpm

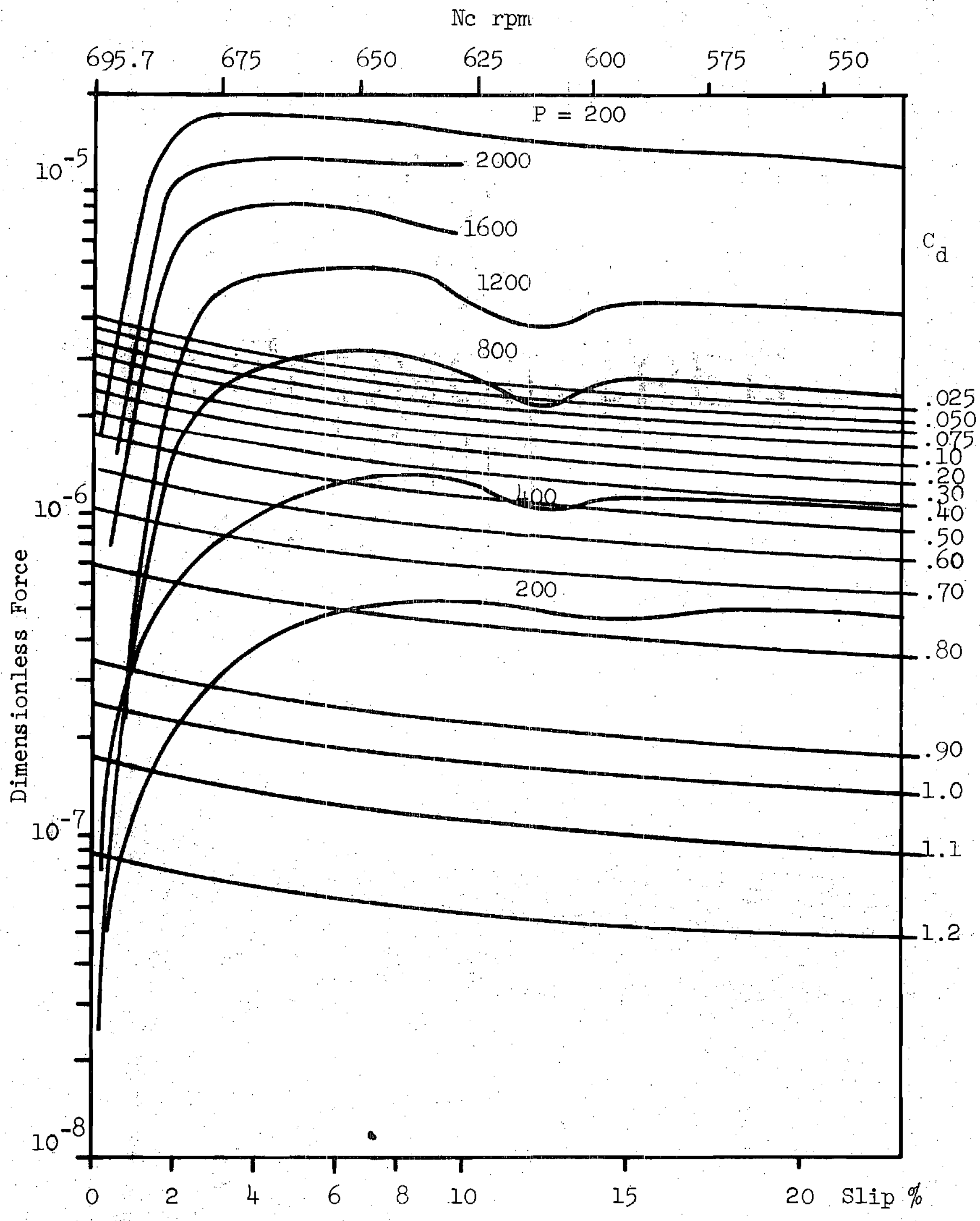

Figure 6. Cage and Drag Forces. Shaft Speed 1750 rpm

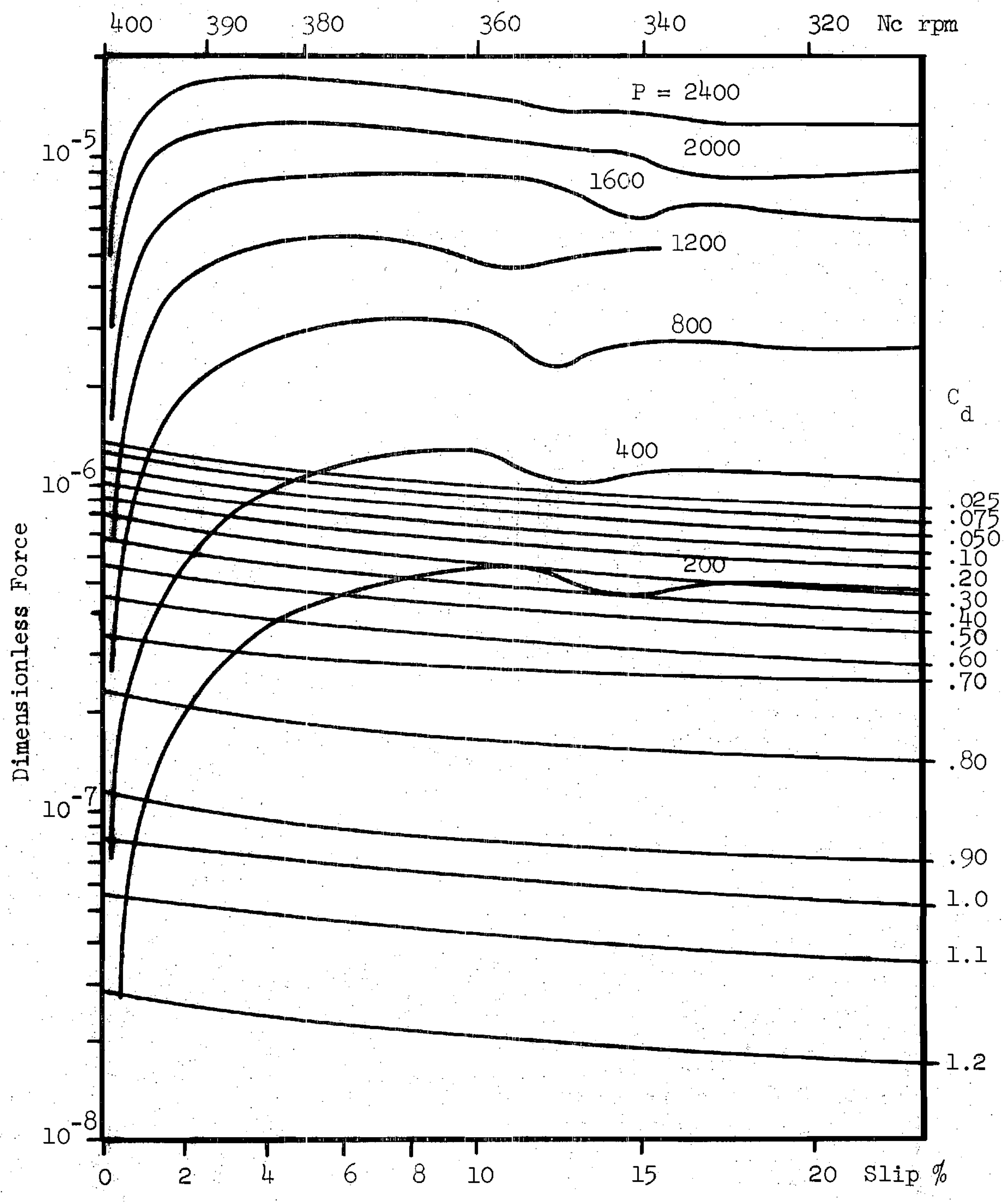

Figure 7. Cage and Drag Forces. Shaft Speed 1006.2 rpm

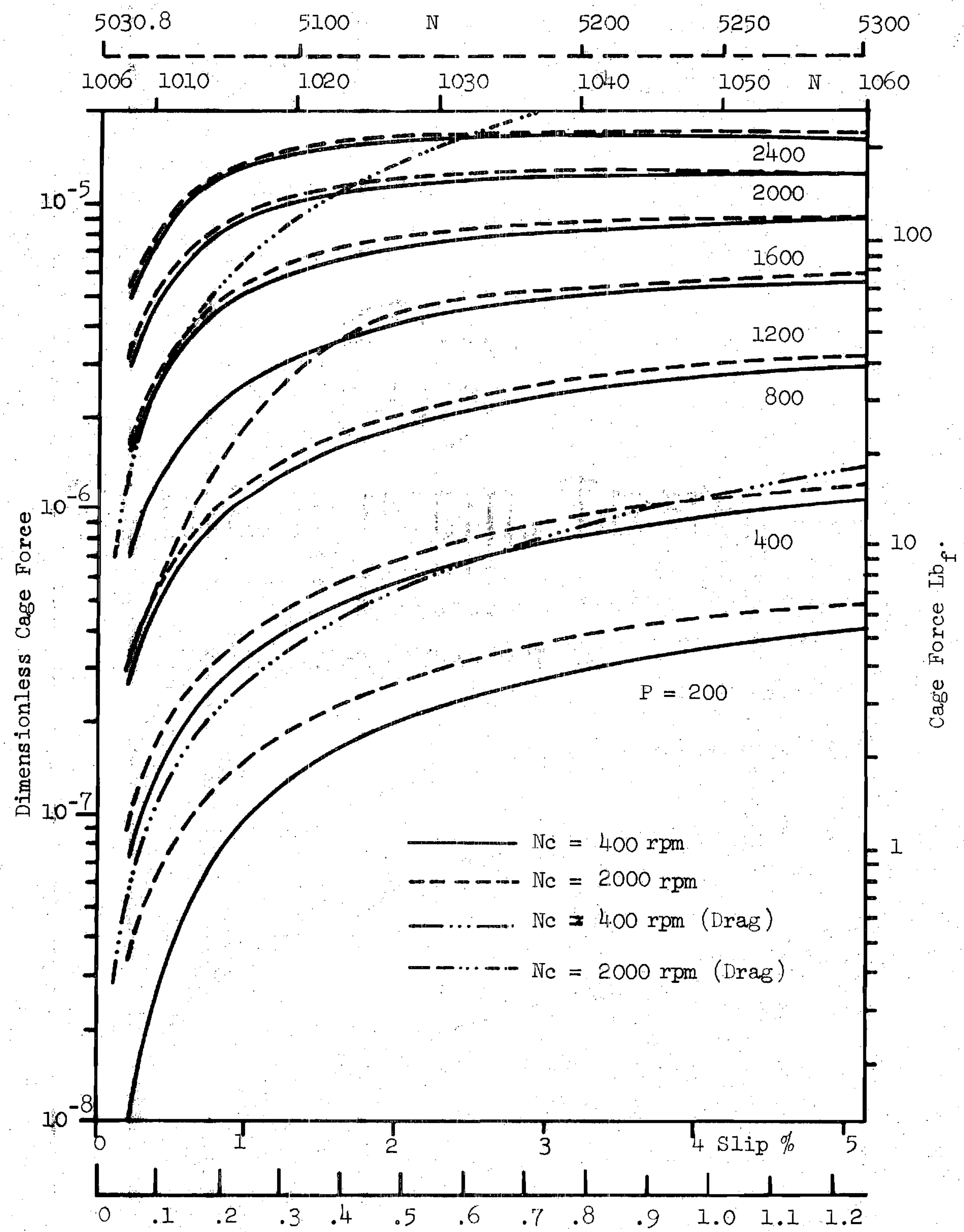

Figure 8. Cage and Drag Forces. Cage Speeds 400 and 2000 rpm

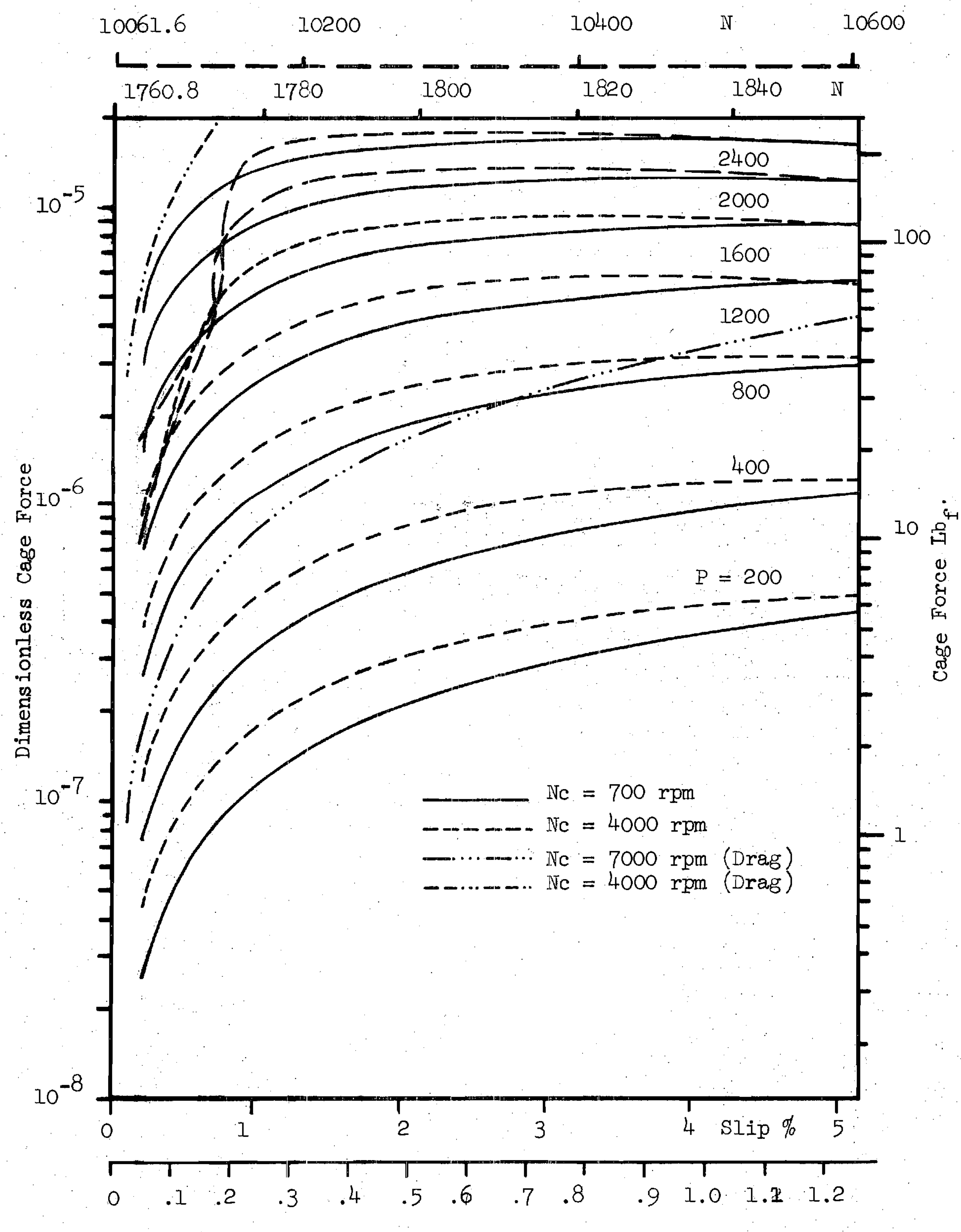

Figure 9. Cage and Drag Forces. Cage Speeds 700 and 4000 rpm

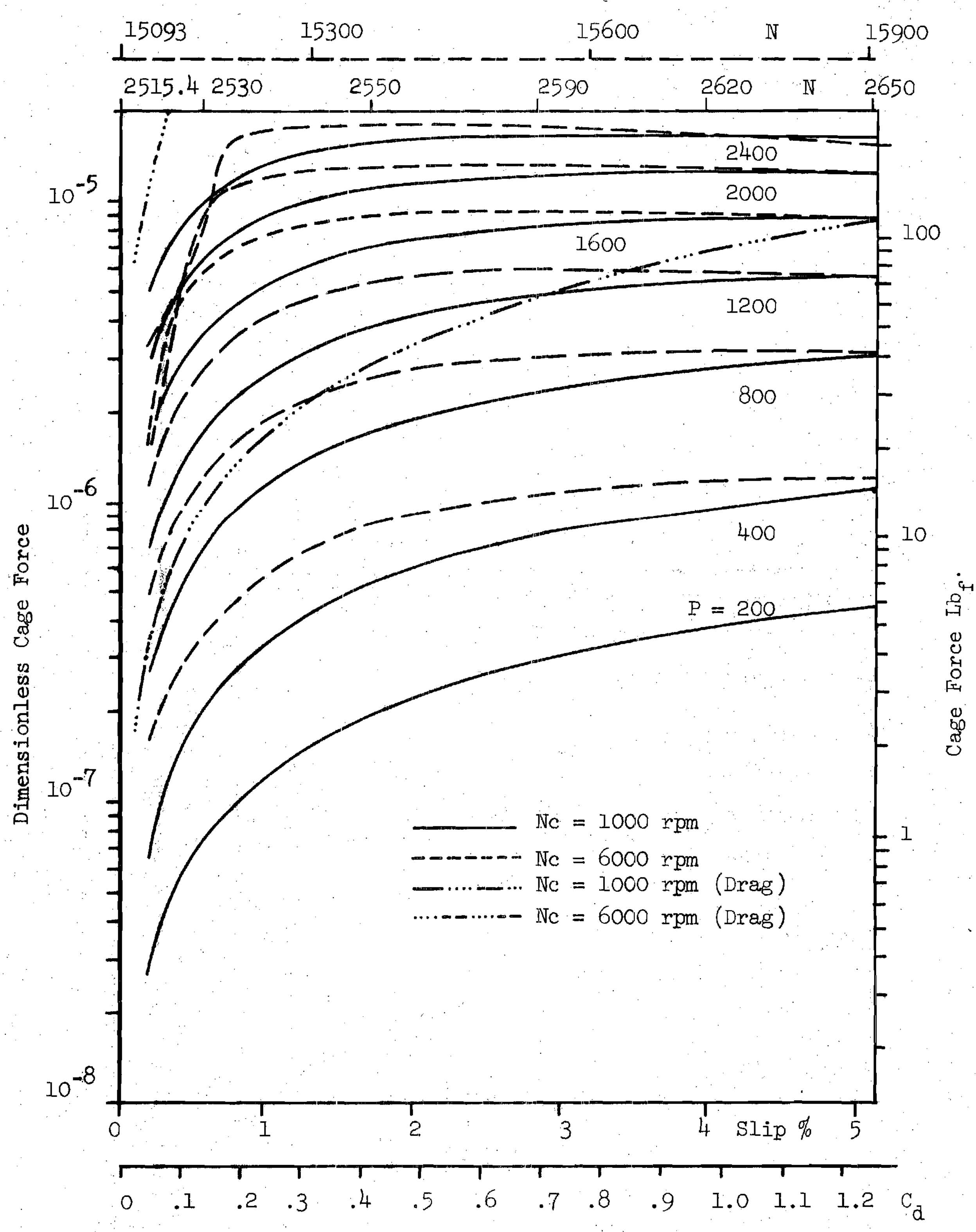

Figure 10. Cage and Drag Forces. Cage Speeds 1000 and 6000 rpm

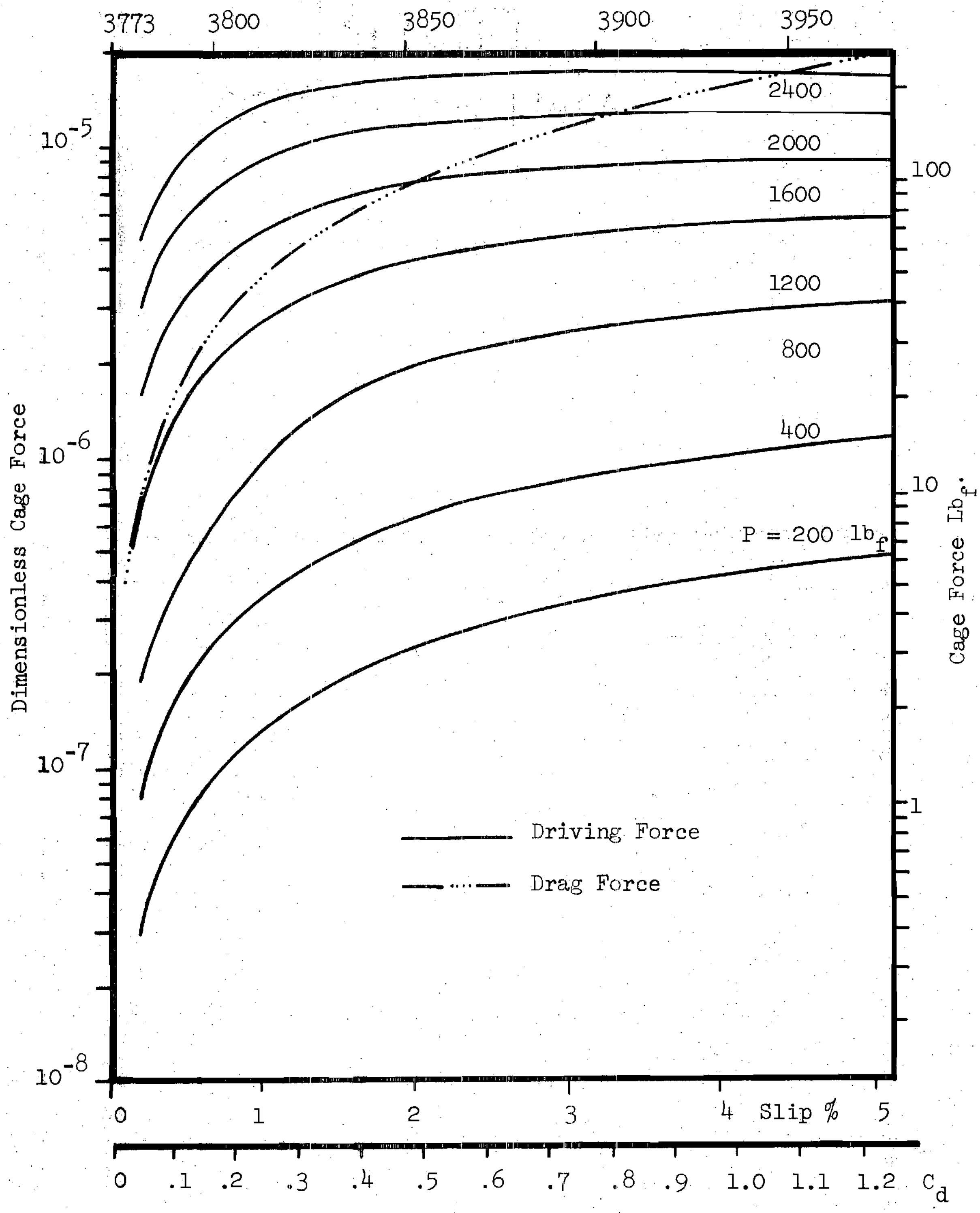

Figure 11. Cage and Drag Forces. Cage Speed 1500 rpm

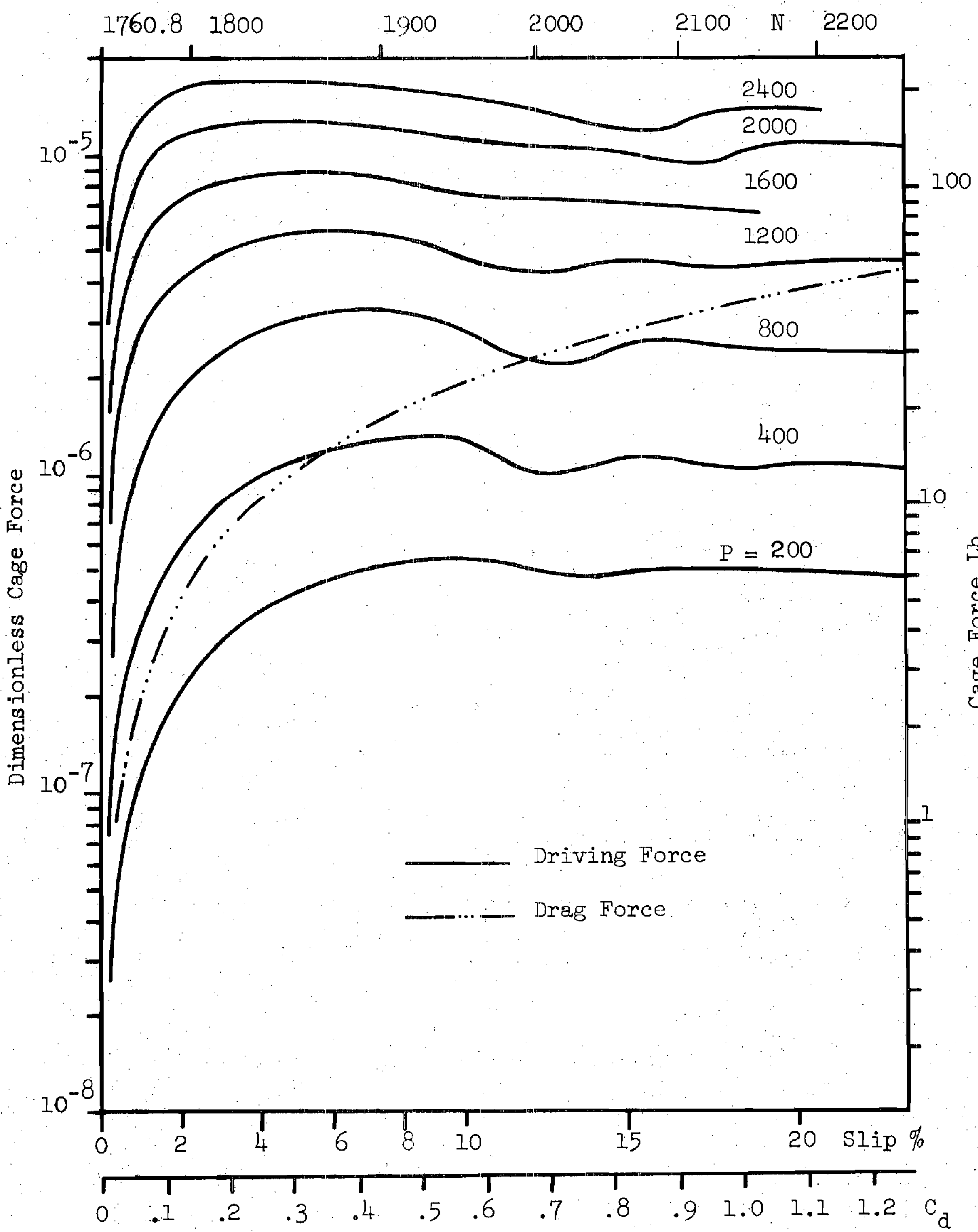

Figure 12. Cage and Drag Forces. Cage Speed 700 rpm

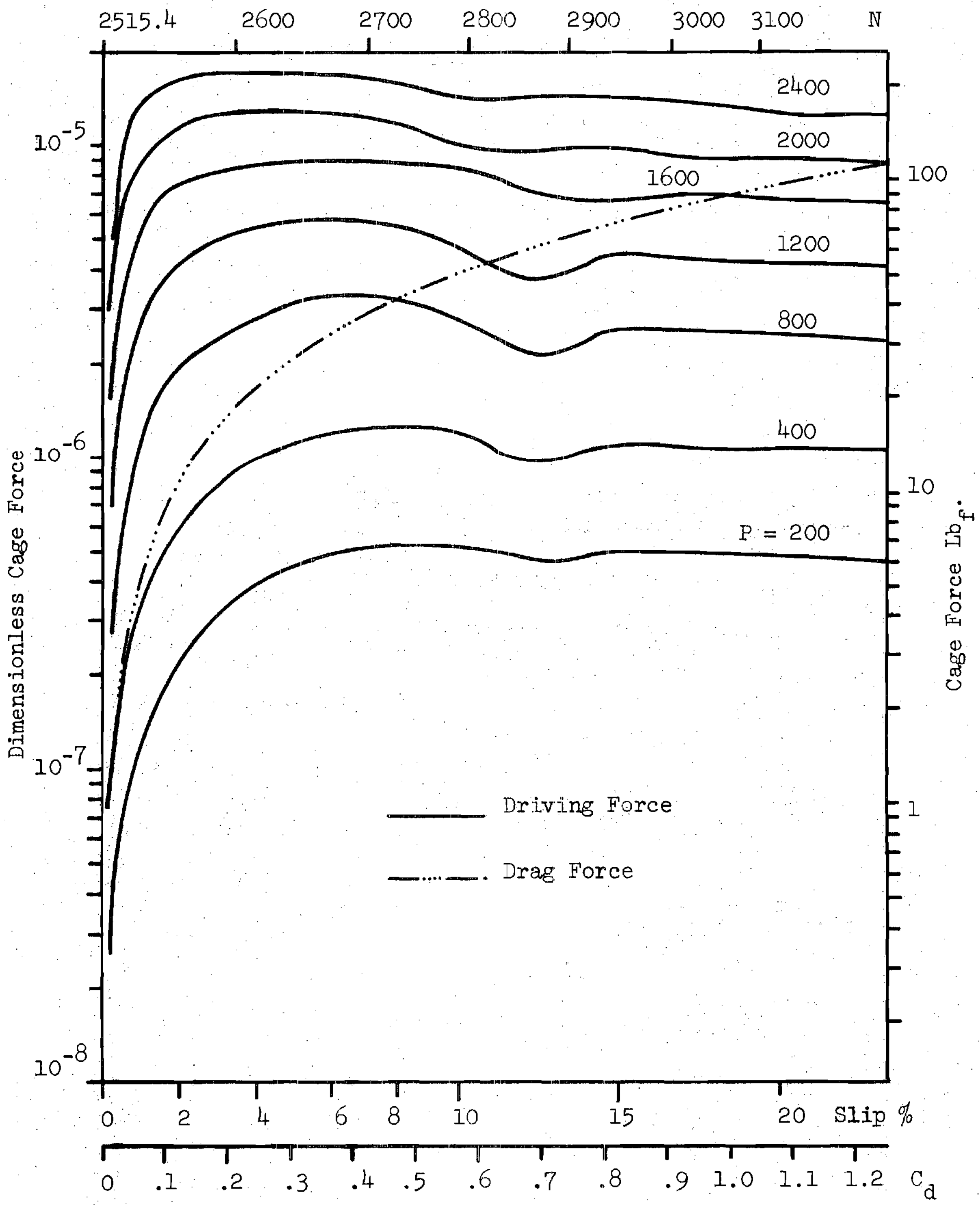

Figure 13. Cage and Drag Forces. Cage Speed 1000 rpm

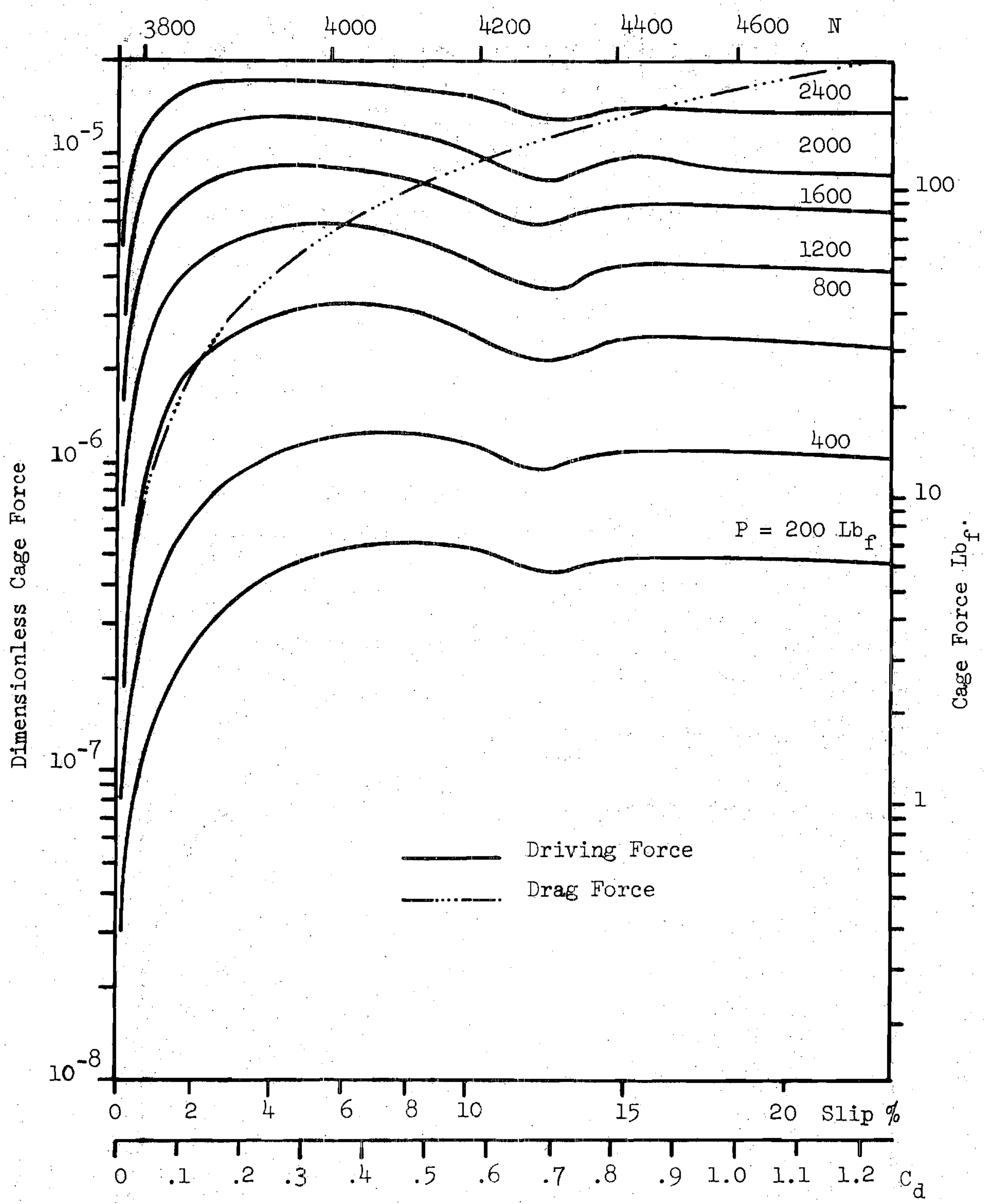

Figure 14. Cage and Drag Forces. Cage Speed 1500 rpm

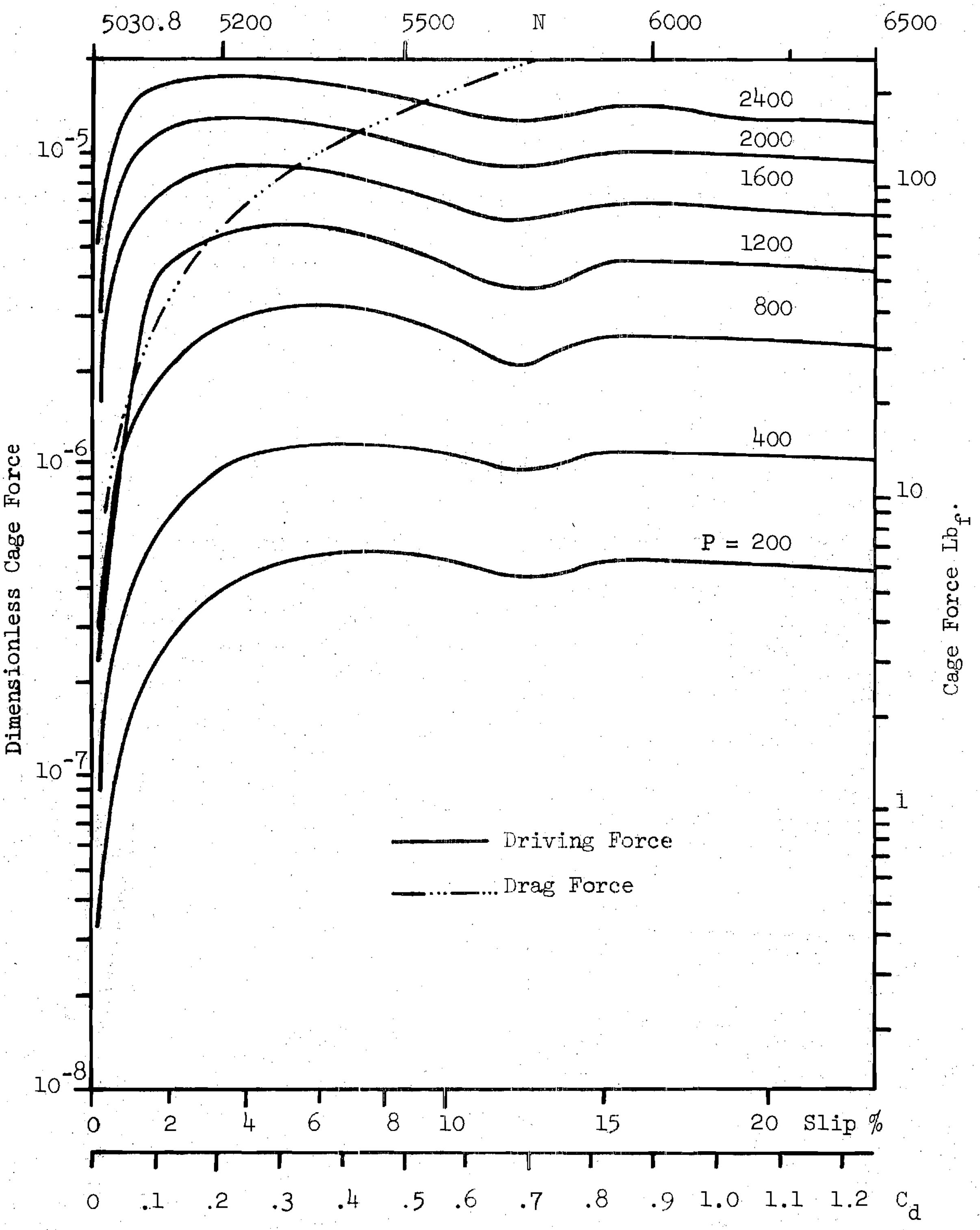

Figure 15. Cage and Drag Forces. Cage Speed 2000 rpm

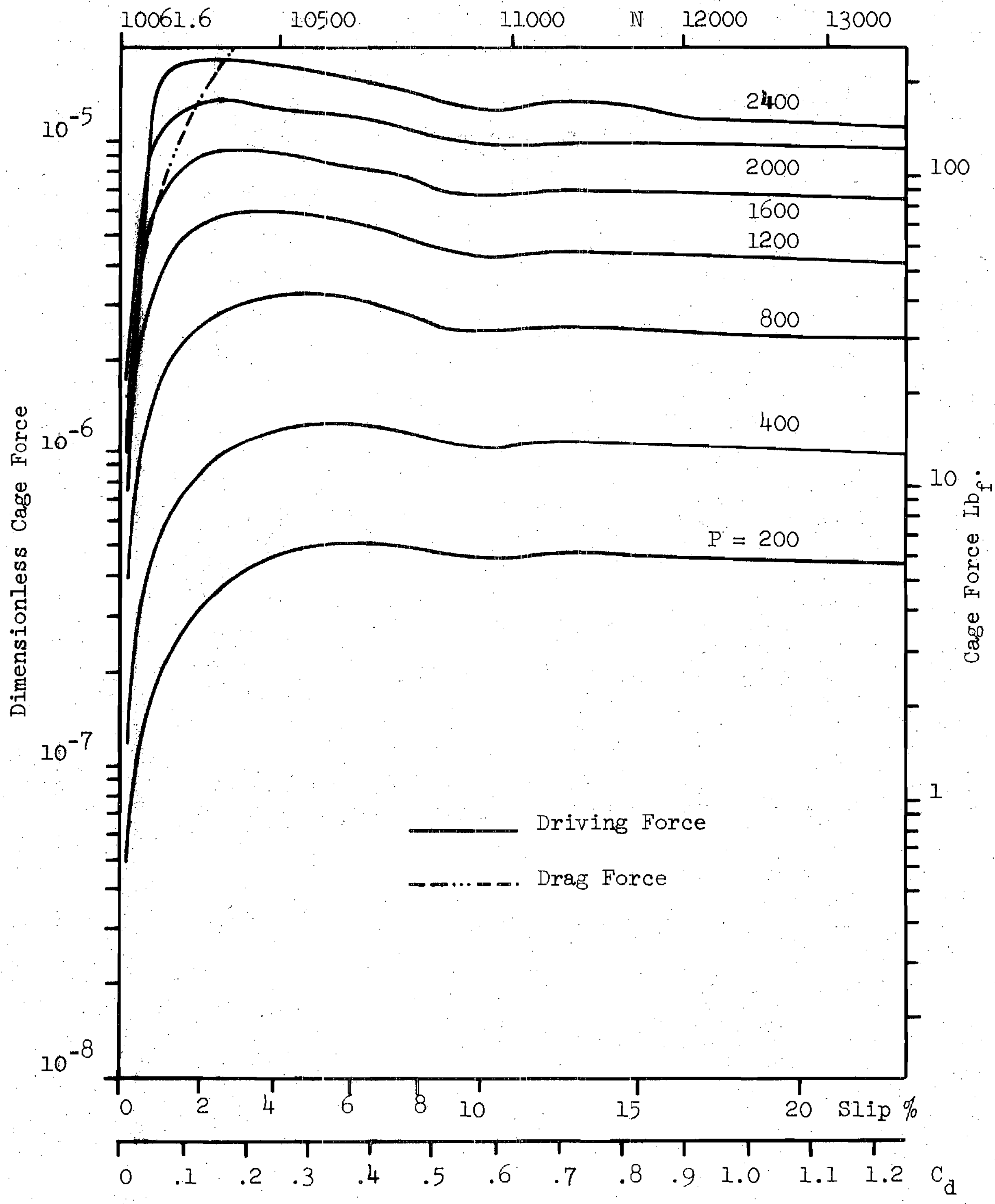

Figure 16. Cage and Drag Forces. Cage Speed 4000 rpm

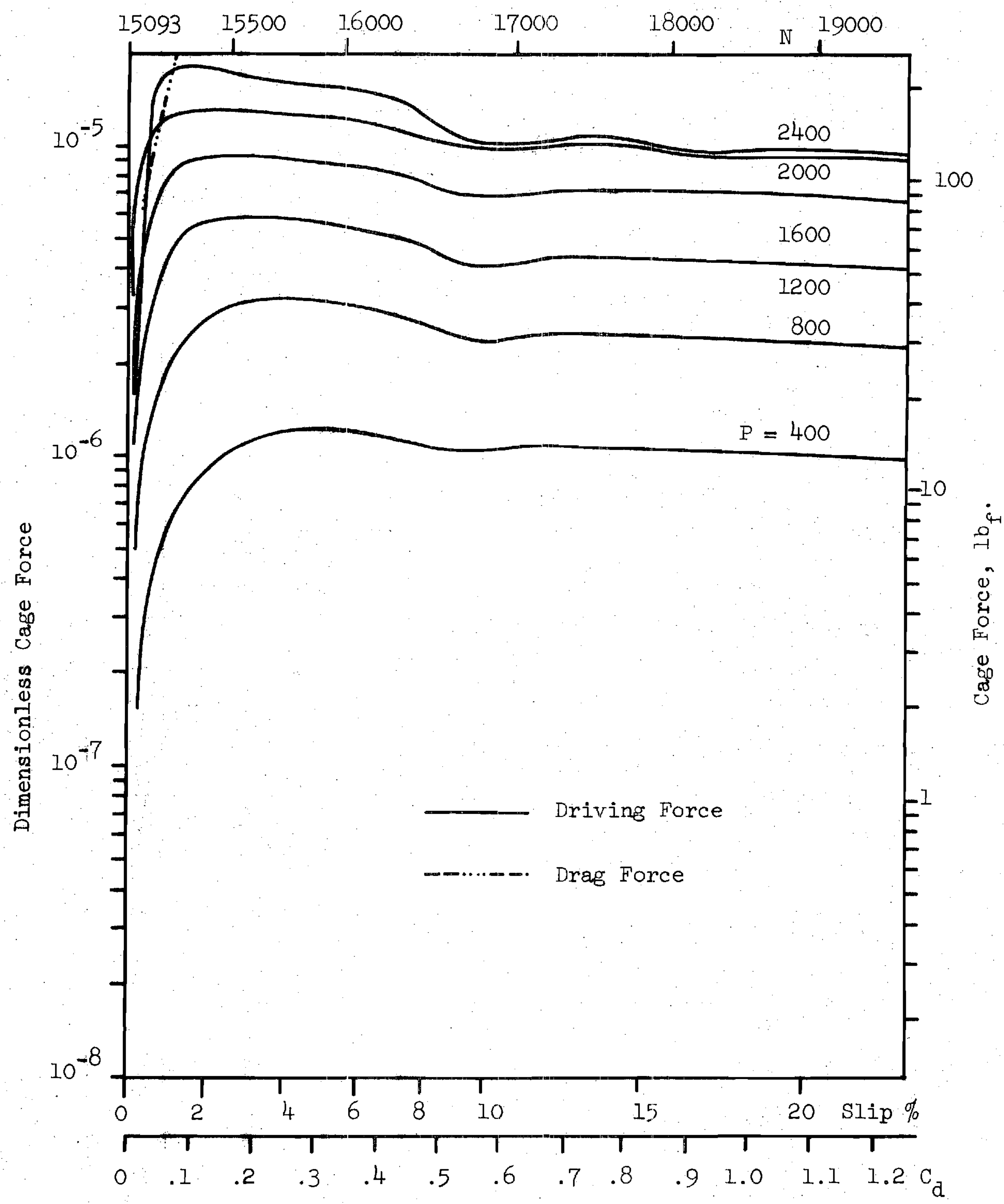

Figure 17. Cage and Drag Forces. Cage Speed 6000 rpm

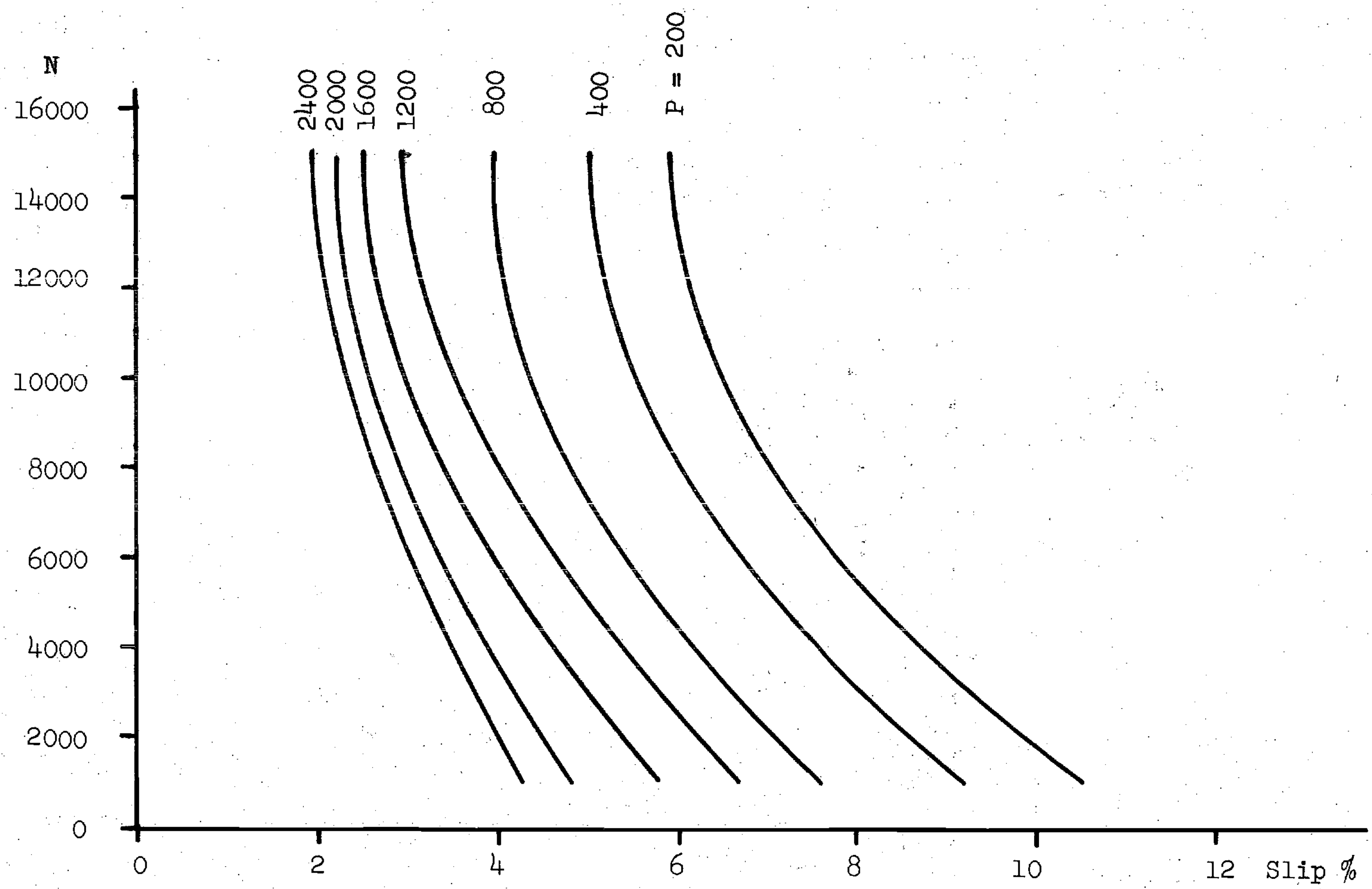

Figure 18. Slip at which Maximum Driving Cage Force is Obtained

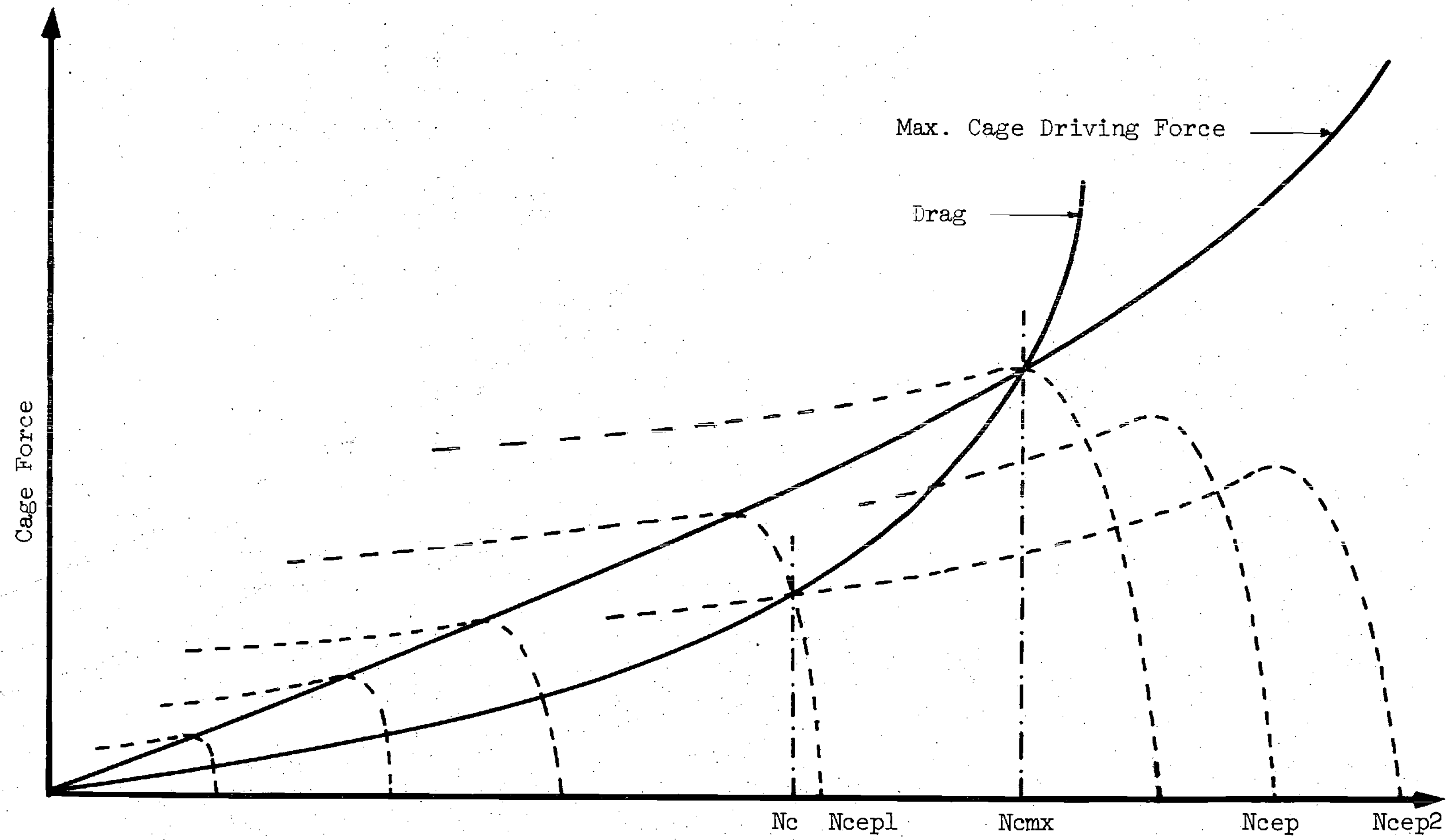

Figure 19. Qualitative Representation of Equilibrium Condition

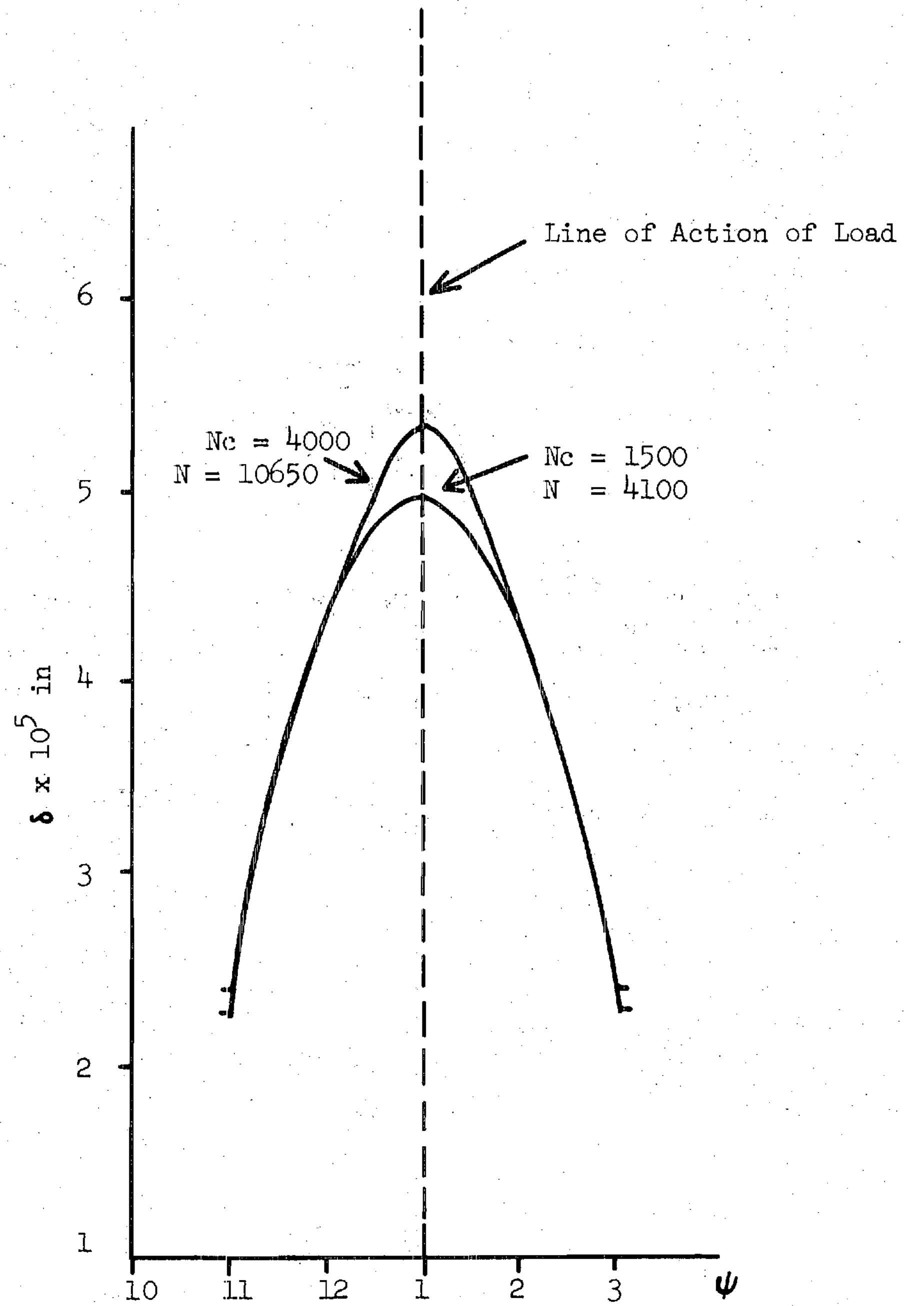

Figure 20. Elastic Deformations at Inner Race Applied Load 400

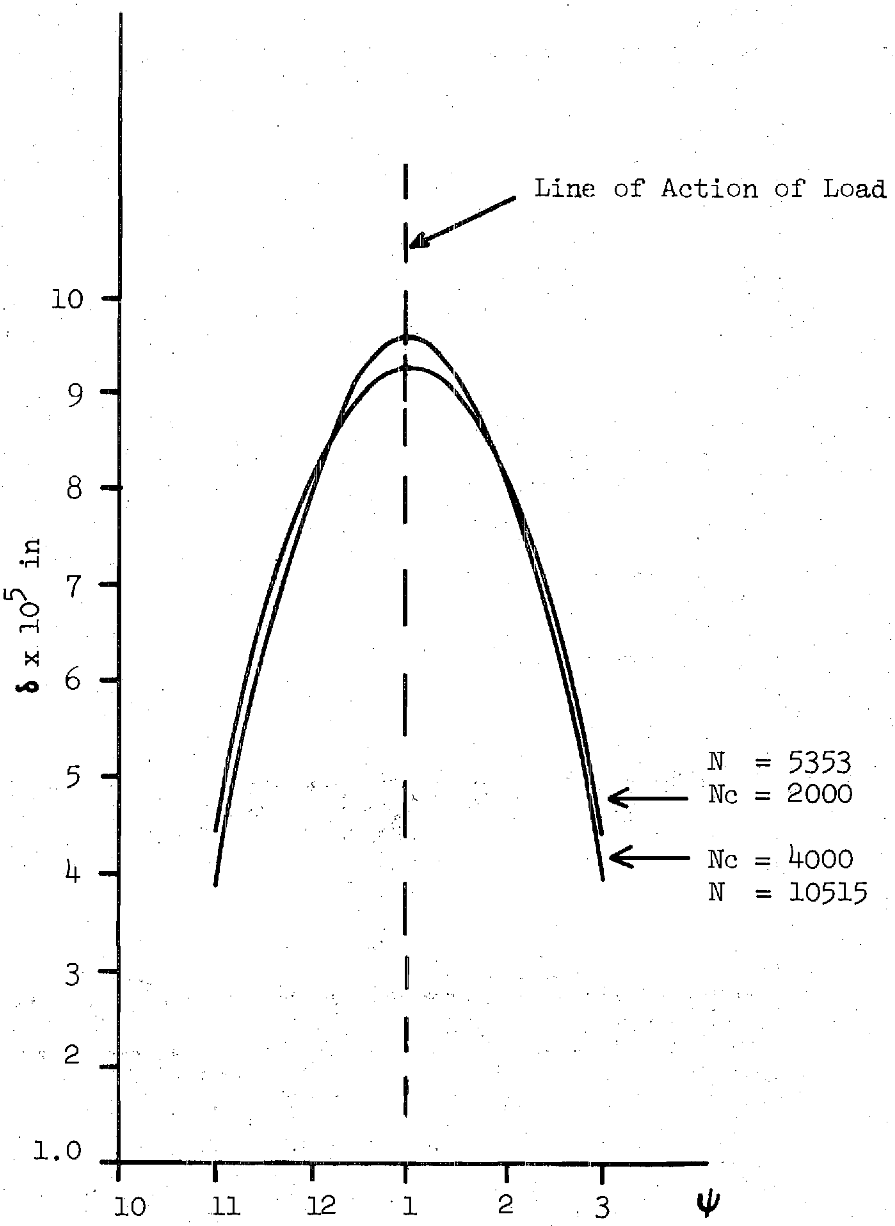

Figure 21. Elastic Deformation at Inner Race Applied Load 800 Lbf.

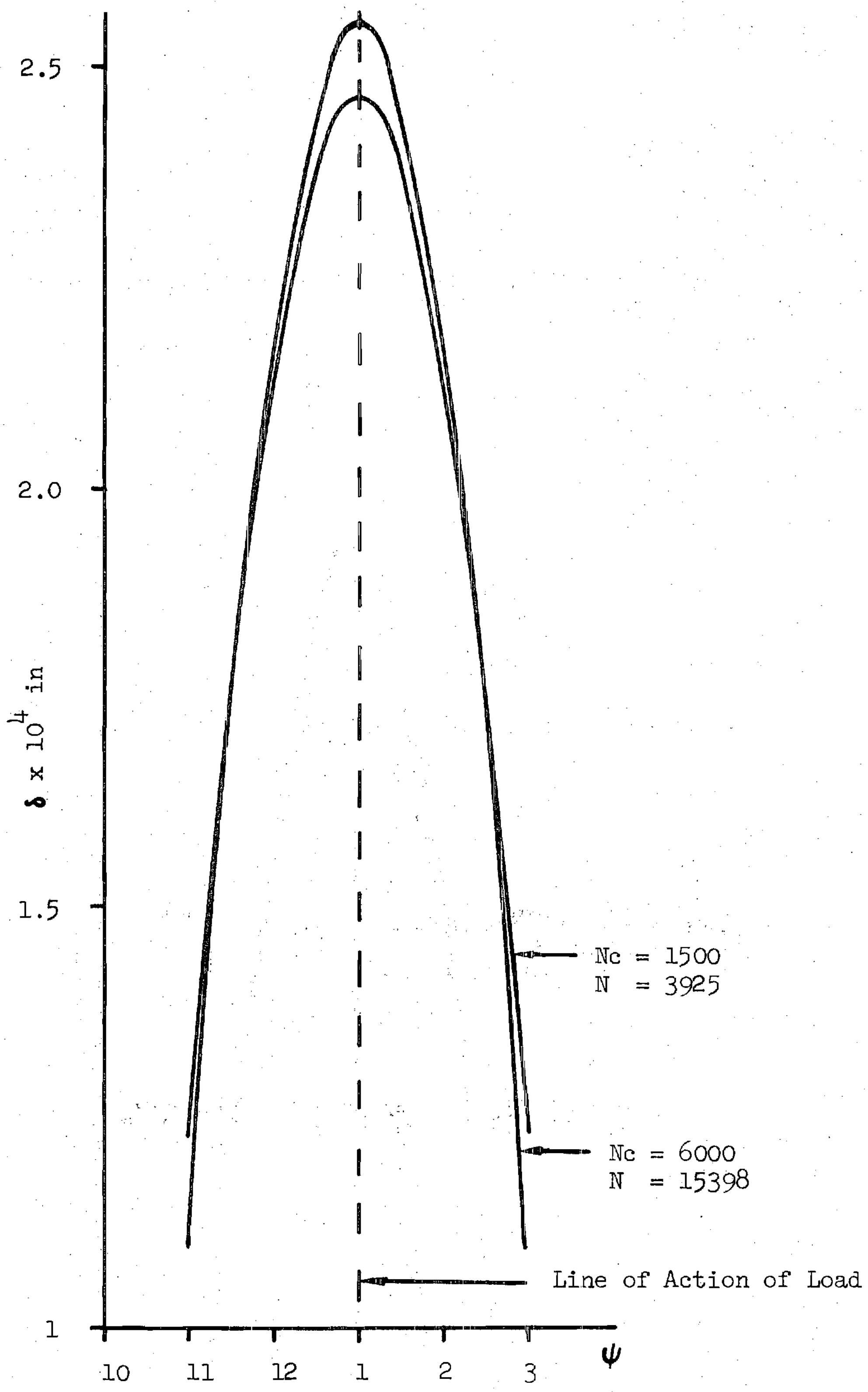

Figure 22. Elastic Deformation at Inner Race Applied Load 2400

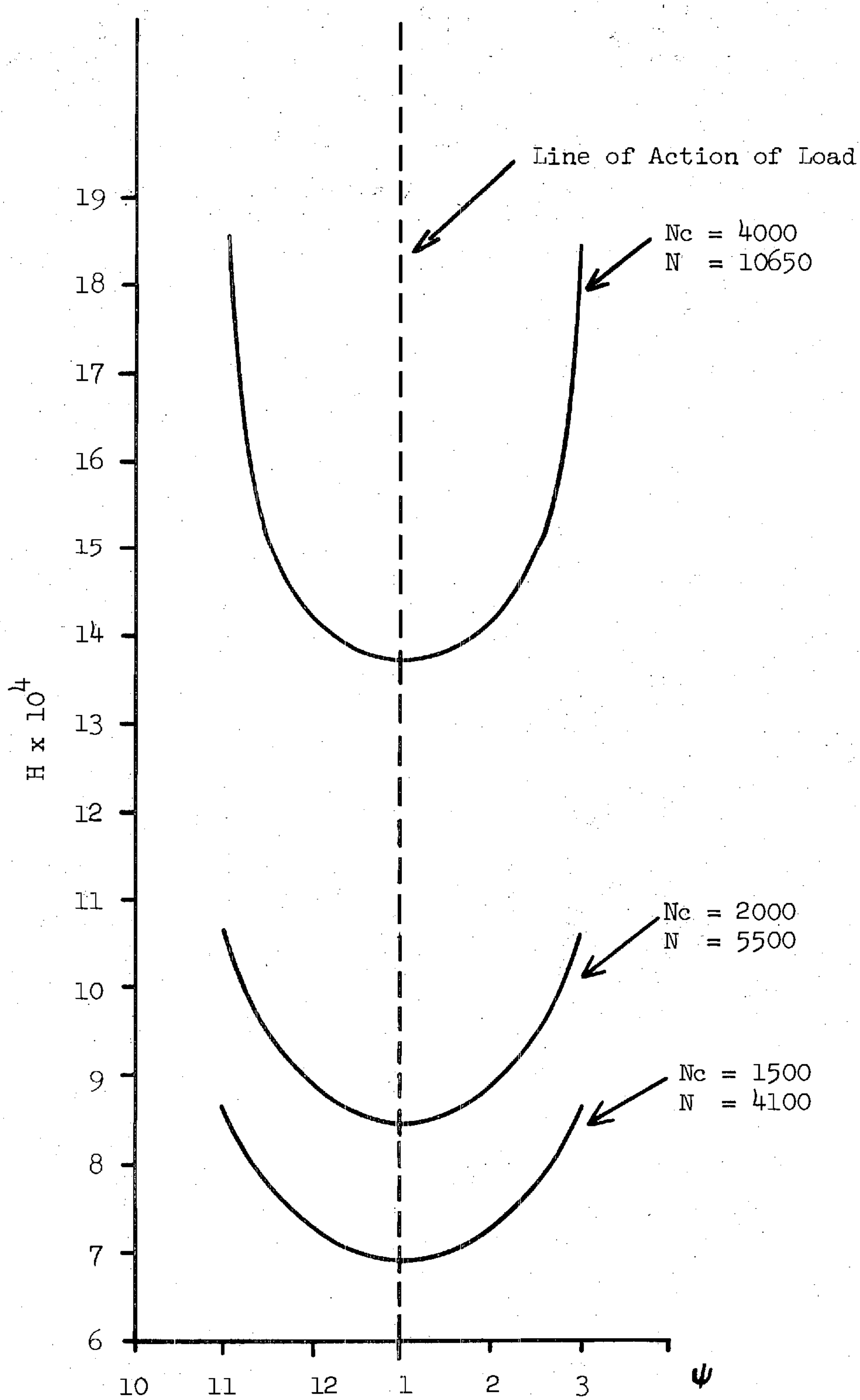

Figure 23. Dimensionless Film Thickness at Inner Race. Applied Load 400 Lb_f.

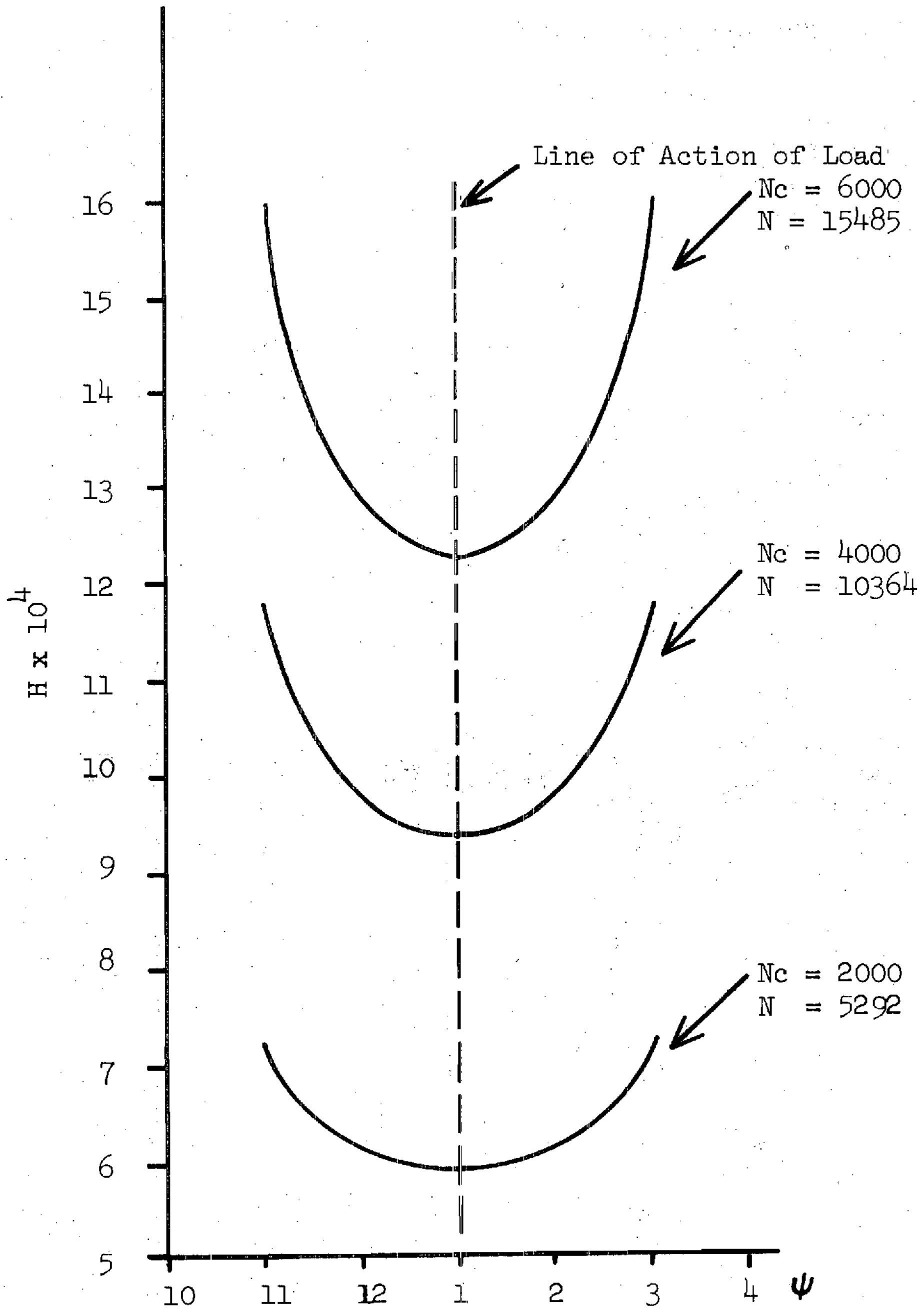

Figure 24. Dimensionless Film Thickness at Inner Race. Applied Load 1600 Lb_f.

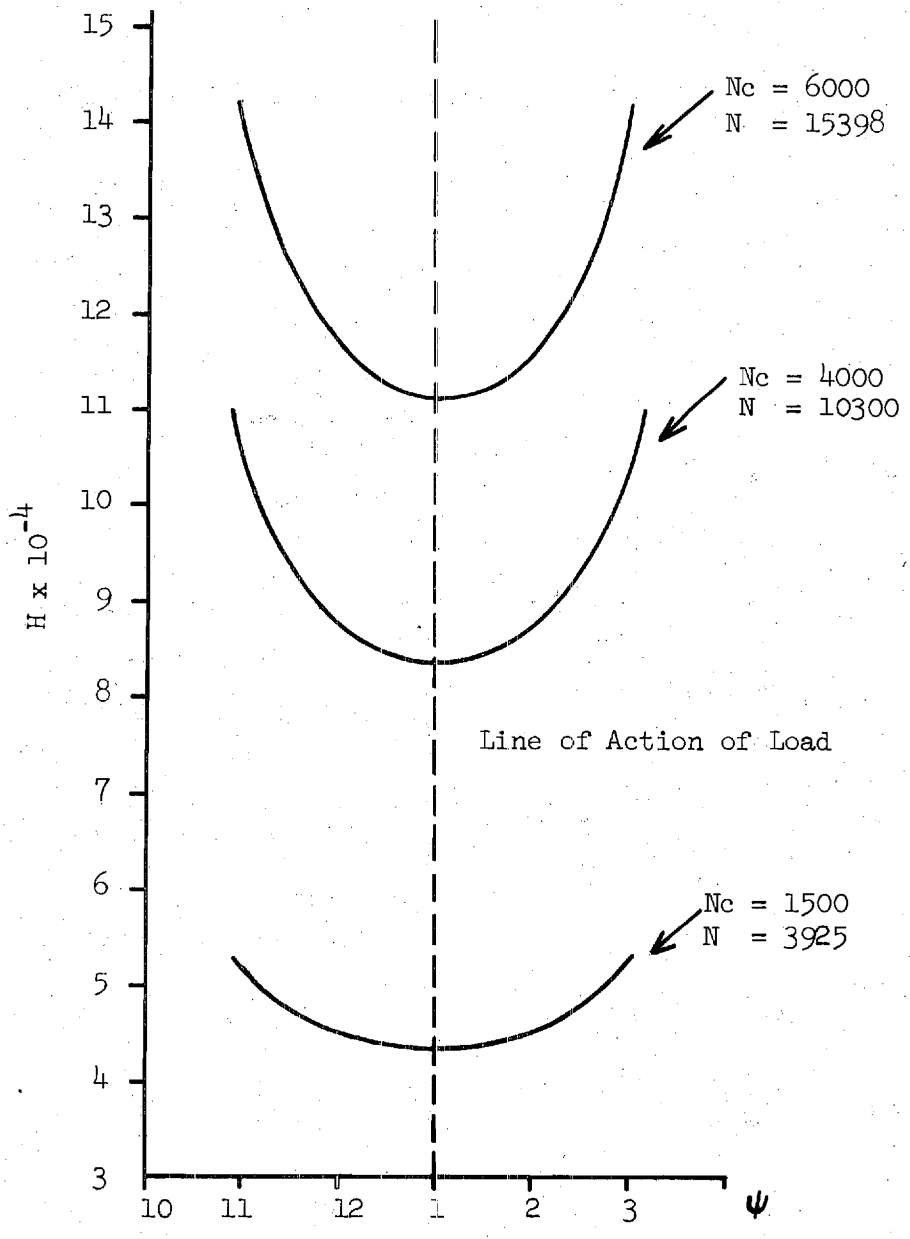

Figure 25. Dimensionless Film Thickness at Inner Race. Applied Load 2400 Lb_f.

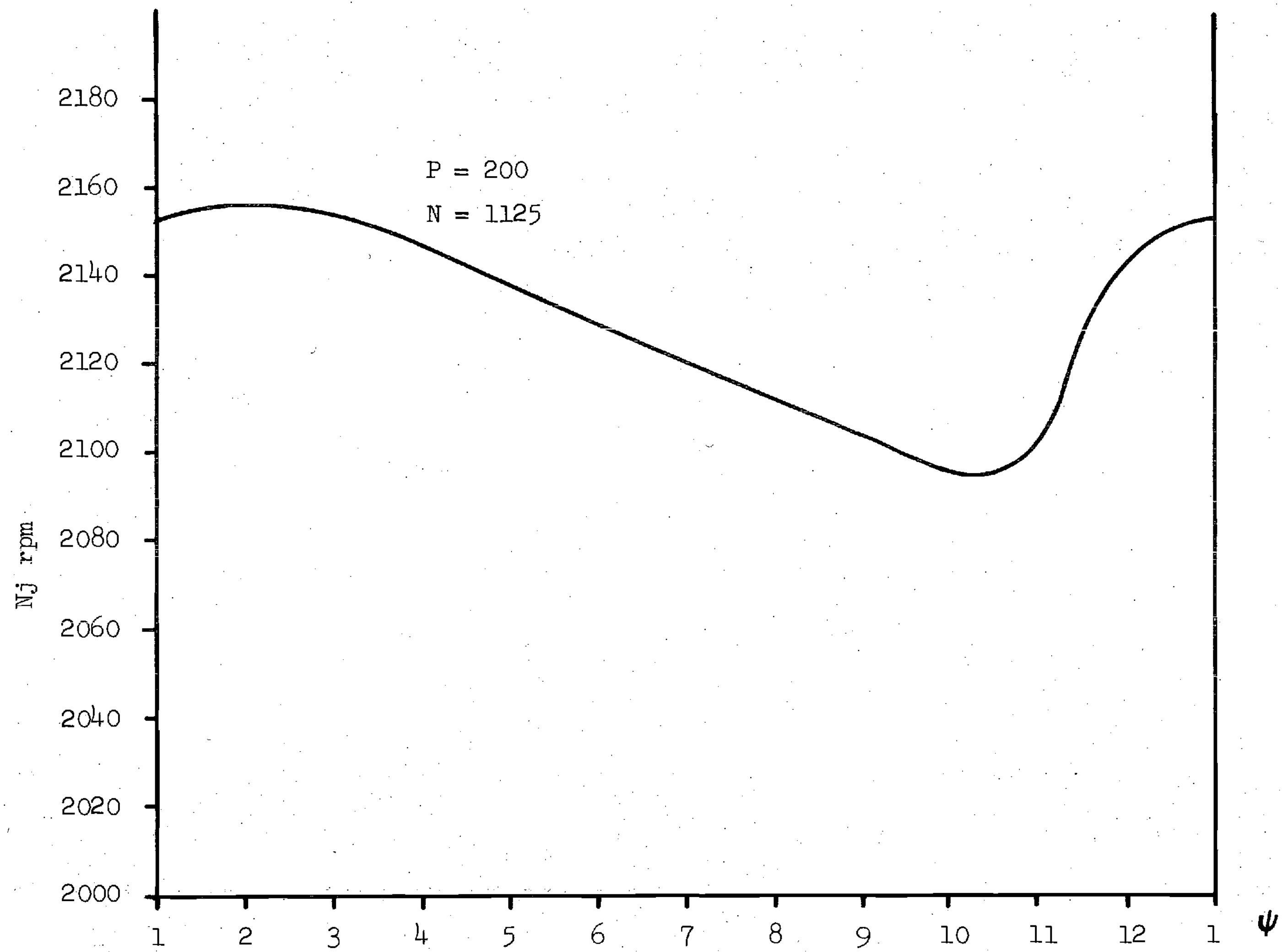

Figure 26. Spinning Speed of Rollers. Cage Speed 400 rpm

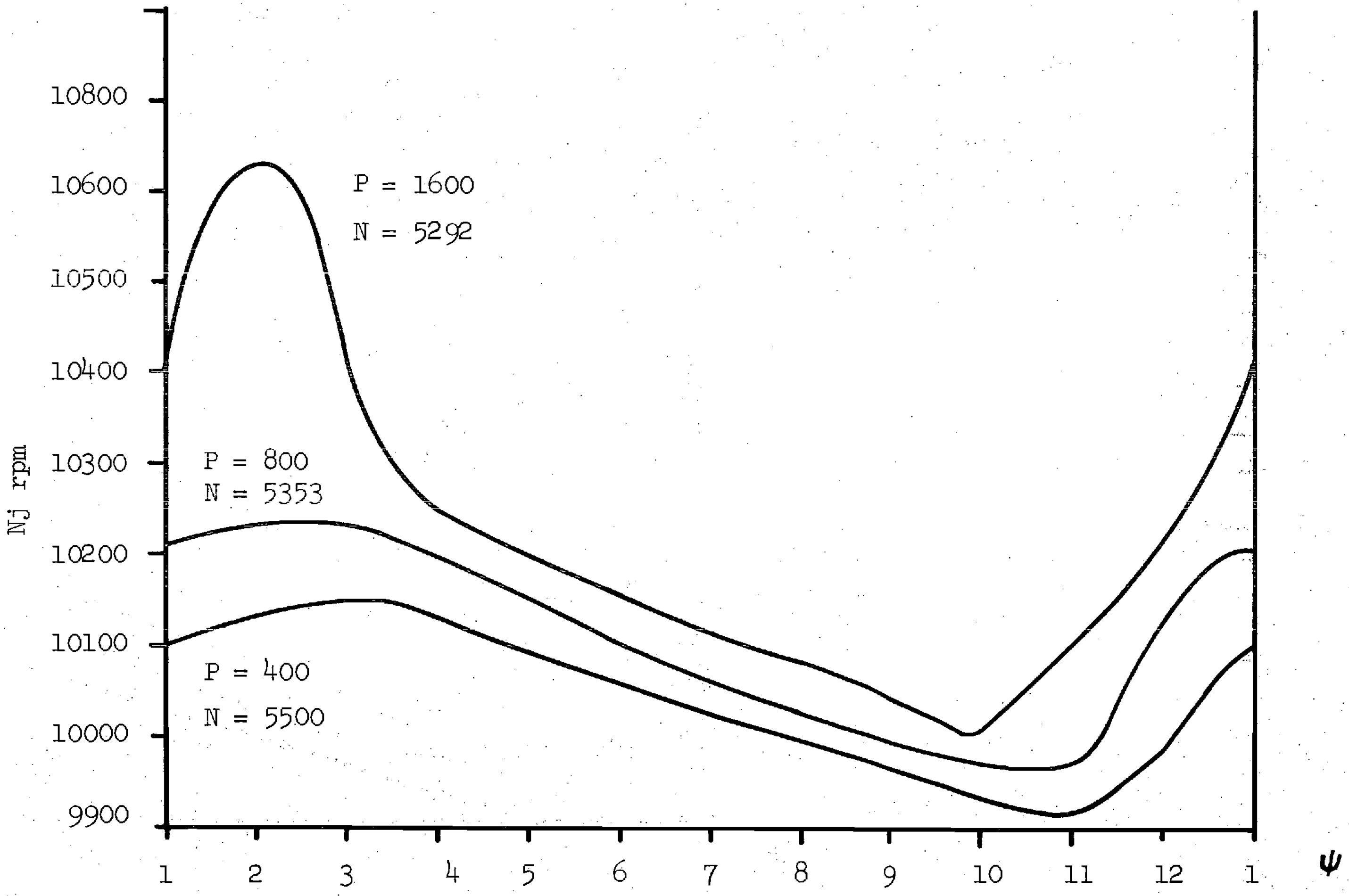

Figure 27. Spinning Speeds of Rollers, Cage Speed 2000 rpm

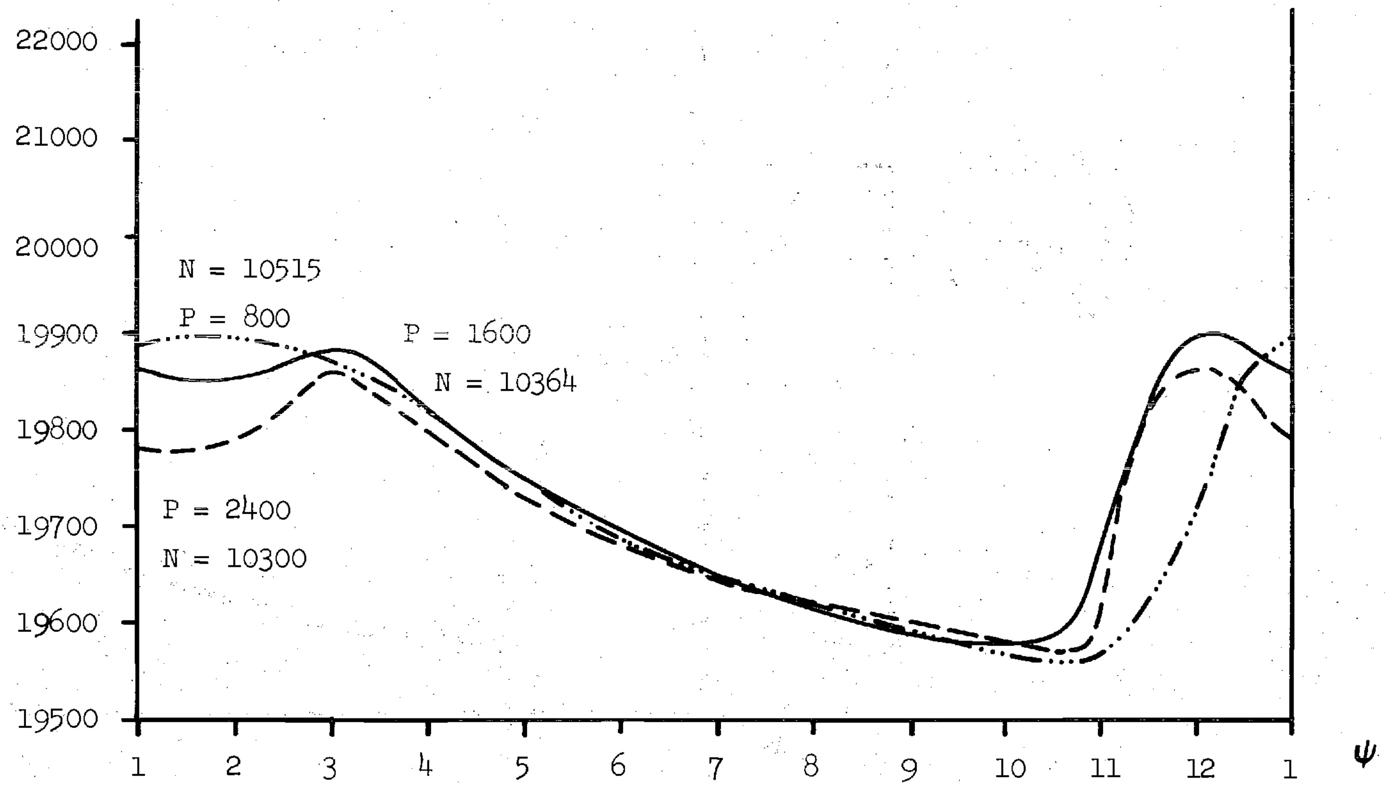

Figure 28. Spinning Speeds of Rollers. Cage Speed 4000 rpm

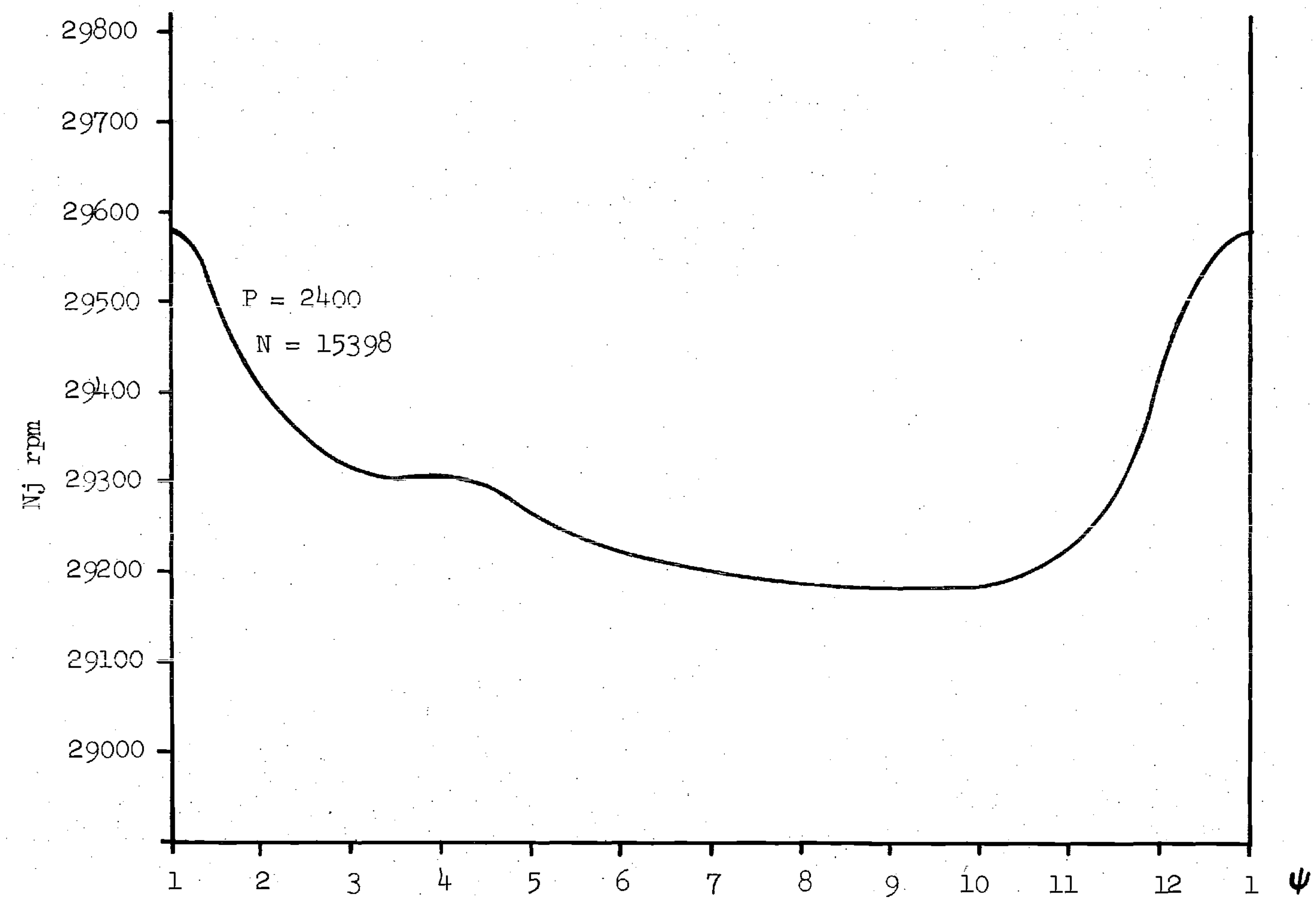

Figure 29. Spinning Speed of Rollers. Cage Speed 6000 rpm

APPENDIX A

PROGRAMS

The required programs in Fortran V are included here. When using them, take into account the comments inserted in each program and make the required changes in parameter and data values.

```
C          DYNAMICS OF ROLLER BEARINGS TAKING INTO ACCOUNT
C                    ELASTOHYDRODYNAMIC FORCES
C
C                                        BY MANUEL A. MOLINA C.
C
C          PROGRAM TO FIND THE FORCES ON THE CAGE FOR GIVEN CAGE SPEED
C          THE FOLLOWING PARAMETERS MUST BE CHANGED WHEN THE NUMBER
C          OF ROLLERS OR THE NUMBER OF ROLLERS IN CONTACT AT THE
C          INNER RACE CHANGE.
C          IZ IS THE TOTAL NUMBER OF ROLLERS IN THE BEARING
C          IZ1 IS THE NUMBER OF ROLLERS IN INNER RACE CONTACT+1
C          IZ3 IS IZ+1 AND IZ2 IS IZ1+1
      PARAMETER IZ=12,IZ1=6,IZ2=7,IZ3=13
C     THIS PROGRAM MUST BE USED WHEN P IS MORE THAN 100
C      OR THE CLEARANCE IS NEGATIVE  AND OF THE ORDER
C      OF 5E-5 IN.
      DIMENSION PHI(IZ1),OD(IZ1,IZ2),PD(IZ1,IZ2)
     1,DEL(IZ1),PL(IZ),PLO(IZ),WO(IZ3),W(IZ3),
     2VI(IZ3),VIO(IZ3),DELNEW(IZ1),DELX(IZ1)
     3,IR(IZ1),JC(IZ1),Q(IZ1,IZ2),PPHI(IZ3)
     4,A(IZ1,IZ2),B(2),DDEL(IZ),DDELO(IZ)
     3,NJ(IZ3),CF(IZ3),SCF(IZ3)
     1,CNJ(241),CVI(241),CVIO(241),CW(241),CCF(241),X(IZ3),D(5)
     1,CAVEL(7),DRAFO(7),DRAG(7)
      COMMON /HERTZ/G,PR,RR,BB
     1/JAT/ANG,PC,CK,P,DELTA,RP,S,CFJ3(40)
     1/CENFOR/G9,NC,C2,C11,C4,C5,AEX,MIU,C6,C7,C8,C9,C10,C1
     2/CCFPNP/C16,ETAO,C13,TO,E,CKFF,PI,RI,RO,LT
      EXTERNAL JATEST,PHERTZ,SYS2,ACDCF,ADINTR,CFJ,PHITER,CFCPNP,FTC
     1,F2N1,FIN
      REAL INH,INS,NC,LT,LE,N,NJ,MIU
      INTEGER Z,Z1,Z2
C          THE FOLLOWING DATA MUST BE CHANGED WHEN REQUIRED,THE
C          SIMBOLS ARE THE SAME AS IN THE THESIS ONLY THE NEXT ARE
C          CHANGED: ETAO IS THE BASIC VISCOSITY,DELES IS THE UNMOUNTED
C          BEARING CLEARANCE,ALPHA IS THE LINEAR COEFICIENT OF
C          THERMAL EXPANSION,AEX IS THE PRESSURE VISCOSITY EXPONENT
C          AND CKFF IS THE THERMAL CONDUCTIVITY OF THE OIL.
      DATA TA,TI,R,RP,LT,LE,ETAO,INH,INS,NC,TO,
     1CO,DELES,E,ALPHA,DH,DS,MIU,AEX,PI,CKFF/100.,190.,.4134,2.0175
     2,.819,.6962,.13185E-3,.6E-3,.85E-3,6000.,150.,1.60
     3,.8E-5,.33E+8,.65E-5,5.5118,2.559065,.05,-.26
     2,3.141592654,.0216/
      IRJ=0
      Z=IZ
      INIT=0
      E7=.001
      ERR9=.09
      CO=1.6*(E*(.852E-4))**.60
      ITE=5
      INT=4
      G=E*(.852E-4)
      S=R/RP
      G9=S
      RI=(1.-S)*S*RP
      C1=(.255E-4)*S**2*LT*RP**3/(LE*E*RI)
      C11=.715*(ETAO*RP*3.141592654/(E*RI*60.))**.71
      RO=(1.+S)*S*RP
      C2=C11*(RO/RI)**.29
      C3=ETAO*3.141592654*RP/(E*RI*30.*CO)
      C4=CO
      C5=RI
```

```
      RJ=.001157*S**4*RP**4*LT
C         RJ IS THE ROLLER MASS MOMENT OF INERTIA
      RJ=1.*RJ
      IF(IRJ)670,670,671
671   RJ=RJ/10.
      XB=0.
      GO TO 672
670   ANCEP=NC
672   C10=1800.*S*RP*LE*E*RI/(3.141592654*RJ)
      C6=RO
      C7=C6*C11/C2
      C8=PI*RP*ETAO/(30.*E*RI)
      C9=C8*RI/RO
      C13=1.43*(ETAO*PI*RP/(60.*E*RI))**.71
      C16=C13*C10
      DELH=INH*(1.+S)*2.*RP/DH
      DELS=INS*DS/(2.*RP*(1.-S))
      DELT=ALPHA*RP*((1.+S)*(TO-TA)-(1.-S)*(TI-TA))
      DELTA=DELES+DELT-DELS-DELH
      CK=(1.14E+7)*LE**(8./9.)
      Z1=Z+1
C         PPHI(J) IS THE ANGULAR POSITION OF ROLLER J
      DO 10 J=1,Z1
   10 PPHI(J)=2.*3.1415927*FLOAT(J-1)/FLOAT(Z)
      DELTA=.0
907   WRITE(6,1)
      READ(5,2,ERR=900,END=900) P,NC
1     FORMAT(' GIVE EXTERNAL LOAD AND CAGE SPEED ')
2     FORMAT()
      II=3
      MIC=II
      II=MIC+IFIX(6.*P/NC)
      IF(II .GT. 20)II=20
      DO 1050 J=1,IZ3
      CF(J)=0.
      SCF(J)=0.
1050  CFJ3(J)=0.
      IF(P .LT. 2.)GO TO 1010
      PRINT 1001
1001  FORMAT(/,/,/,/,'        NC            P            EPNC          CSLIP
     1    N        CAGE FORCE',/,/)
      PC=(.225E-4)*S**2*RP**3*LT*NC**2
C         THE CALCULATION OF THE GUESSED INTIAL VALUES OF THE
C         ELASTIC DEFORMATIONS BEGINS NOW.
      N=2.*NC/(1.-S)
      EPNJ=NC/S
      DNN=N/500.
      DO 1000 MAN=1,25
      N=N+DNN
      IF(MAN .GT. 15)N=N+5.*DNN
      IF(MAN .GT. 19)N=N+10.*DNN
      IF(MAN .GT. 24)N=N+50.*DNN
      INIT=0
      XB=0.
      IF(MAN .GT. 1)GO TO 673
      DELPC=(PC/CK)**.9
      DELTA1=DELTA+2.*DELPC
      FAC=3.5
      IF(P .GT. 1500.)FAC=3.6
      IF(P .LT. 320.)FAC=2.16
      DO 19 J=1,Z
      DL=((FAC*(P+CF(J))/(CK*FLOAT(Z)))**.9)*(ABS(COS(PPHI(J))))**1.5
      IF(J .EQ. 1)ARRR=DL
```

```
      IF(DL .LE. .0) DL=.0
        (
        DELTA1) 17,17,18
17    DDEL(J)=DL-DELTA1+ABS(COS(.5+PPHI(J)))/5.
C          DEEL ARE THE ELASTIC DEFORMATIONS AT THE INNER CONTACT
      GO TO 19
   18 DIN=DELTA1*COS(PPHI(J))/5.
      IF(DIN .LT. 0.) DIN=.0
      DDEL(J)=DL+DIN
   19 CONTINUE
      DDEL(Z1)=DDEL(1)+ARRR+DELTA/2.+DELPC
      IT=50
      L2=0
      NSZ=Z
      NIZ=IFIX(ACOS(DELTA1/(2.*DDEL(Z1)))/PPHI(2))
      L1=10
      NZ=2*NIZ+1
C          NZ IS NUMBER OF ROLLERS IN CONTACT AT INNER RACE
      IF(NZ+1-IZ1)200,201,200
200   PRINT 202,NZ
202   FORMAT(' NUMBER OF ROLLERS IN CONTACT IS NOT IZ1, NZ=',I2)
      GO TO 900
201   DO 877 J=1,NIZ
      DEL(J)=DDEL(J)
  877 DEL(NZ-J+1)=DDEL(J+1)
      DEL(NZ+1)=DDEL(Z+1)
      CFJ3(1)=CF(1)
      DEL(NIZ+1)=DDEL(NIZ+1)
      PHI(1)=PPHI(1)
  601 DO 991 J=1,NIZ
      PHI(J+1)=PPHI(J+1)
      CFJ3(J+1)=CF(J+1)
      CFJ3(NZ-J+1)=CFJ3(J+1)
  991 PHI(NZ-J+1)=PHI(J+1)
      PHI(NZ+1)=PHI(1)
      CFJ3(NZ+1)=CFJ3(1)
      L2=L2+1
      ANG=PPHI(2)
      Z=NZ
      Z1=Z+1
      DO 85 K=1,Z
   85 PD(K,1)=PC/CK
      PD(Z+1,1)=-P/CK
      DO 20 K=1,Z
      DO 30 J=2,Z1
      IF((J-1) .EQ. K) PD(K,J)=1.
      IF((J-1) .NE. K) PD(K,J)=.0
   30 CONTINUE
   20 PD(K,Z+2)=-1.
      DO 40 J=2,Z1
   42 PD(Z+1,J)=COS(PHI(J-1))
   40 CONTINUE
      PD(Z+1,Z+2)=1.0
      Z2=Z+2
      EPS1=(.1E-35)
      IERR1=0
      IERR2=0
      EPS3=.0001*DEL(1)
      IRR=0
C          THE FOLLOWING SUBROUTINE GIVES THE DEFORMATIONS AT THE INNER
C          CONTACT,DELNEW ARE THE DESIRED ROOTS.
```

```
      CALL NSIMEQ(JATEST,PD,CD,A,Z1,IR,JC,Z1,Z2,
     10,25,SI,DS,SF,EPS1,EPS2,EPS3,0,DEL,DELNEW,DELX,IERR1,IERR2)
      IF(IERR1 .EQ. -1) IRR=3
      IF(IERR2 .EQ. -1) IRR=2
      IF(IRR-2) 903,902,901
  903 NIZN=IFIX(ACOS(DELTA1/(2.*DELNEW(Z1)))/PHI(2))
      IF(NIZN-NIZ) 909,908,909
  909 WRITE(6,119) NIZN
  119 FORMAT(' THE NUMBER OF ROLLERS IN CONTACT IS 2*NIZ+1 ,NIZ =',I3)
     GO TO 900
  908 Z=NSZ
      DDEL(1)=DELNEW(1)
      NIZ2=NIZ+1
      DO 111 J=1,NIZ
      DDEL(J+1)=DELNEW(J+1)
  111 DDEL(Z-J+1)=DELNEW(J+1)
      DO 600 J=1,NIZ2
      BETA=ASIN(DELNEW(Z1)*SIN(PHI(J))/(RP*(1.+S)))
      DDELO(J)=DELNEW(Z1)*COS(PHI(J))-DELTA/2.-DDEL(J)
     1+RP*(1.+S)*(1.-COS(BETA))
  600 IF(DDELO(J) .LT. 0.) DDELO(J)=0.
   50 DO 988 J=1,NIZ
  988 DDELO(Z-J+1)=DDELO(J+1)
      DDEL(Z+1)=DELNEW(Z1)
      NIZ1=NIZ+2
      NIZ2=Z-NIZ
      DO 113 J=NIZ1,NIZ2
      DDELO(J)=(PC/CK)**0.9
  113 DDEL(J)=0.
      DO 117 J=1,Z
  117 PHI(J)=PPHI(J)
      IWRMX=0
      DO 100 J=1,Z
      IF(DDEL(J)) 299,299,300
  300 PL(J)=CK*DDEL(J)**1.11
      GO TO 703
  299 PL(J)=0.
  703 IF(DDELO(J)) 302,302,301
  301 PLO(J)=CK*DDELO(J)**1.11
      GO TO 702
  302 PLO(J)=0.
  702 PVER=PL(J)*COS(PHI(J))
      W(J)=PL(J)/(E*RI*LE)
      IF(W(J) .GT. (.1E-5))IWRMX=1
      WO(J)=PLO(J)/(E*RO*LE)
100   IF(WO(J) .GT. (.1E-5))IWRMX=1
      W(IZ3)=W(1)
      IF(IWRMX .NE. 1)II=7
      IF(P-300.)704,704,705
705   IWRMX=1
      GO TO 620
704   DO 137 J=1,Z
      IF(W(J)-W(J-1))3,4,3
    4 VI(J)=VI(J-1)
      GO TO 5
    3 IF(W(J)) 38,38,35
   35 PR=(W(J)/(2.*3.141595))**.5
      BB=4.*PR*RI
      B(1)=-4.*PR
      B(2)=-B(1)
      FK=1.
      RR=RI
```

```
      VI(J)=SIM3NI(PHERTZ,B,.01,.TRUE.,100,1.,$39)
      GO TO 5
   39 PRINT 8,J
    8 FORMAT(' INTEGRAL DOES NOT CONVER.,J=',I4)
      GO TO 5
   38 VI(J)=0.
    5 IF(WJ(J)-WO(J-1))6,7,6
    7 VIO(J)=VIO(J-1)
      GO TO 137
    6 IF(WO(J)) 136,136,135
  135 PR=(WO(J)/(2.*3.141595))**.5
      GPR=G*PR
      B(1)=-4.*PR
      RR=R0
      B(2)=-B(1)
      BB=B(1)*R0
      VIO(J)=SIM3NI(PHERTZ,BB,.01,.TRUE.,100,1.,$130)
      GO TO 137
  130 PRINT 140,J
  140 FORMAT(' VIO DOES NOT CONVERGE,J=',I3)
      GO TO 137
  136 VIO(J)=0.
  137 CONTINUE
      WO(IZ3)=WO(1)
      W(IZ3)=W(1)
      VI(IZ3)=VI(1)
      VIO(IZ3)=VIO(1)
      GO TO 620
  901 WRITE (6,9)
      GO TO 900
  902 WRITE(6,11)
   31 FORMAT()
    9 FORMAT('PROGRAM ENDED BY ERROR IERR1=-1')
   11 FORMAT('PROGRAM ENDED BECAUSE IERR2=-1')
      GO TO 900
  620 IF(INIT)602,603,602
603   DO 604 KA=1,IZ3
      NJ(KA)=NC/S+ABS(((1.-S)/(2.2*S))*DNN*COS(.5*PPHI(KA)))
      IF((NJ(KA)-NC/S) .LT. 0.)NJ(KA)=NC/S
      IF(IWRMX-1)640,641,640
641   CALL CFCPNP(CF(KA),ACC,DCF,N,NJ(KA),W(KA))
      GO TO 639
640   CALL ACDCF(ACC,DCF,N,NJ(KA),VI(KA),VIO(KA),W(KA),CF(KA))
639   CF(KA)=CF(KA)*LE*E*RI*ABS(NJ(KA))*MIU/NJ(KA)
      IF(CF(KA)-(.1E+5))604,642,642
642   PRINT 31,CF(KA)
      GO TO 900
 604  CONTINUE
602   DO 618 LI=1,IZ3
      IF(MIU .LT. (.1E-10))GO TO 618
      CF(LI)=CF(LI)/(LE*E*RI)*ABS(NJ(LI))/(NJ(LI)*MIU)
618   SCF(LI)=CF(LI)
      I3=IZ*II+1
      IF(INIT)673,688,673
688   DO 680 J=1,IZ3
680   X(J)=FLOAT(J-1)/FLOAT(IZ)
      DX=1./FLOAT(IZ*II)
      XB=0.
      DO 681 J=1,I3
      IF(J .GT. (3*II) .AND. J .LT. (I3-3*II))GO TO 682
      CNJ(J)=GNINT(X,NJ,IZ3,3,XB,D,$683)
      CVI(J)=GNINT(X,VI,IZ3,3,XB,D,$683)
```

```
      CVIO(J)=GNINT(X,VIO,IZ3,3,X' ,D,$683)
      CW(J)=GNINT(X,W,IZ3,2,XB,D,$683)
      IF(CW(J) .LT. 0.)CW(J)=0.
      CCF(J)=GNINT(X,CF,IZ3,3,XB,D,$683)
      GO TO 681
C         IN THE PREVIOUS INTERPOLATING SUBROUTINES IT MUST BE NOTED
C         THAT NJ IS THE SET OF INITIAL VALUES OF ROLLER SPEEDS,W ARE
C         THE DIMENSIONLESS INNER CONTACTS LOADS,CF ARE THE ESTIMATED
C         FORCES OF THE ROLLERS ON THE CAGE,VI AND VI0 ARE THE
C         INTEGGRALS REQUIRED TO CALCULATE THE TRACTION FORCES
C         THE HIGGINSON'S METHOD IS USED.
682   CNJ(J)=STINT(X,NJ,IZ3,3,XB,D,$683)
      CVI(J)=STINT(X,VI,IZ3,3,XB,D,$683)
      CVI0(J)=STINT(X,VI0,IZ3,3,XB,D,$683)
      CW(J)=STINT(X,W,IZ3,2,XB,D,$683)
      IF(CW(J) .LT. 0.)CW(J)=0.
      CCF(J)=STINT(X,CF,IZ3,3,XB,D,$683)
681   XB=XB+DX
673   CALL SYS2(N,CNJ,CVI,CVI0,CW,CCF,IZ,I3,ITE,E7,IER,IWRMX,II,CAF0)
      IF(IER .NE. -1)IRJ=IRJ+1
      IF(IRJ .LT. I .AND. IER .NE. -1)GO TO 671
      GO TO 685
683   PRINT 686
686   FORMAT(' CONTINGENCY IN INTERPOLATING SUBROUTINE')
      GO TO 900
685   DO 687 J=1,I3,II
      KK=J/II+1
      NJ(KK)=CNJ(J)
687   CF(KK)=CCF(J)
      IF(IER+1) 605,606,605
606   PRINT 607,IER
607   FORMAT(' DIVERGING SYS2,IER=',I3)
      GO TO 900
605   IF(MIU) 608,614,608
608   ER8=0.
      ER9=0.
      DO 609 K=1,IZ3
      IF(SCF(K)) 610,611,610
610   ER8=(CF(K)-SCF(K))/SCF(K)
      GO TO 609
611   IF(CF(K))613,609,613
613   ER8=(CF(K)-SCF(K))/CF(K)
609   IF(ABS(ER8) .GT. ER9)ER9=ABS(ER8)
      IF(ER9-ERR9)614,614,615
615   DO 616 K=1,IZ3
616   CF(K)=CF(K)*LE*E*RI*(ABS(NJ(K))/NJ(K))*MIU
      INIT=INIT+1
      IF(INIT-INT)601,601,621
C         THIS RETURN TO 601 TO INTRODUCE THE FORCES OF THE CAGE ON
C         THE ROLLERS,DUE TO FIRCTION,IN THE LOAD DISTRIBUTION.
621   PRINT 622,ER9
622   FORMAT(' GENERAL ITERATION DIVERGING, ER9=',())
614   EPNJ=(1.-S)*N/(2.*S)
      EPNC=N*(1.-S)/2.
      CSLIP=NC/EPNC-1.
      PRINT 731,NC,P,EPNC,CSLIP,N,CAF0
731   FORMAT(1X,5(E11.6,1X),E11.6)
1000  CONTINUE
      GO TO 907
1010  DIVA=LE*E*RI
      CAVEL(1)=400.
      CAVEL(2)=700.
```

```
      CAVEL(3)=1000.
      CAVEL(4)=1500.
      CAVEL(5)=2000.
      CAVEL(6)=4000.
      CAVEL(7)=6000.
      DO 1008 I=1,7
      RE=4.593*RP**2*CAVEL(I)
      CN=3.87/(RE**.5)
      IF(RE .GT. 300000.)CN=.146/(RE**.2)
      DF=(.1239E-6)*CN*CAVEL(I)**2*RP**4.
      REI=4.593*RP*(1.-S)*CAVEL(I)/50.
      FI=16./REI
      IF(REI .GT. 2500.)FI=(48./REI)*(REI/2500.)**.85596
      REO=2.*.04593*RP*CAVEL(I)*(1.+S)
      FO=16./REØ
      IF(REO .GT. 2500.)FO=(48./REO)*(REO/2500.)**.85596
      DAI=(.7786E-5)*LT*FI*CAVEL(I)**2*(1.-S)**3.*RP**3.
      DAO=(.7786E-5)*LT*FO*RP**3.*(1.+S)**3.*CAVEL(I)**2
1008  DRAG(I)=(DAO+DAI+DF)/DIVA
      DRACØ=.0
      AREA=4.*PI*RP*(S*RP)
      CONST=(.113E-3)*AREA*(2.*PI*RP/60.)**2
      CØNST=CØNST/DIVA
      PRINT 1031,CAVEL
      PRINT 1032,DRAG
      PRINT 1034
1031  FØRMAT(' NC  =',7E9.4,/)
1032  FØRMAT(' REF 4 =',7E9.4)
      DO 1111 L=1,15
      DRACØ=DRACO+.025
      IF(L .GT. 4)DRACØ=DRAC0+.075
      DO 1112 K=1,7
1112  DRAFO(K)=CONST*DRACØ*CAVEL(K)**2
1111  PRINT 1033,DRACØ,DRAFØ
1033  FØRMAT(8E9.4)
1034  FØRMAT(1X,/,' DRAG CØEF    DRAG FØRCES AT EACH NC....',/)
      GO TØ 907
900   END
409:>
```

```
C           DYNAMICS OF ROLLER BEARINGS TAKING INTO ACCOUNT
C                      ELASTOHYDRODYNAMIC FORCES
C
C                                        BY MANUEL A. MOLINA C.
C
C           THIS PROGRAM CALCULATES THE CAGE FORCE,SPINNING SPEED
C           DISTRIBUTION,SLIPS,FILM THICKNESSES,EPICYCLIC SPEEDS,LOADS
C           AND DEFORMATIONS FOR  GIVEN CAGE AND SHAFT SPEEDS
C           WHEN  THE PROPER SUBROUTINE SYS2 IS USED.
C           THE PROGRAM ALSO CAN CALCULATE THE PREVIOUS QUANTITIES
C           AFTER FINDING THE EQUILIBRIUM OPERATING CONDITION OF THE
C           BEARING FOR A GIVEN CAGE SPEED AND A GUESSED SHAFT SPEED.
C            THE SHAFT SPEED IS CHANGED BY THE PROGRAM UNTILL THE EQUILI-
C           BRIUM CONDITION IS FOUND.IN THIS CASE THE SECOND FORM OF SYS2

MUST BE
C           USED. SEE THE COMMENTS IN SYS2.
C           SIMBOLS AND COMMENTS OF THE FIRST MAIN PROGRAM ALSO
C           APPLIES TO THIS PROGRAM.
      PARAMETER IZ=12,IZ1=6,IZ2=7,IZ3=13
C     THIS PROGRAM MUST BE USED WHEN P IS MORE THAN 100
C      OR THE CLEARANCE IS NEGATIVE  AND OF THE ORDER
C      OF 5E-5 IN.
      DIMENSION PHI(IZ1),OD(IZ1,IZ2),PD(IZ1,IZ2)
     1,DEL(IZ1),PL(IZ),PLO(IZ),WO(IZ3),W(IZ3),TC(IZ3),
     2VI(IZ3),VIO(IZ3),DELNEW(IZ1),DELX(IZ1)
     3,IR(IZ1),JC(IZ1),Q(IZ1,IZ2),PPHI(IZ3)
     4,A(IZ1,IZ2),B(2),DDEL(IZ),DDELO(IZ)
     3,NJ(IZ3),CF(IZ3),SCF(IZ3),H(IZ3),SLIP(IZ3)
     1,CNJ(241),CVI(241),CVIO(241),CW(241),CCF(241),X(IZ3),D(5)
      COMMON /HERTZ/G,PR,RR,BB
     1/JAT/ANG,PC,CK,P,DELTA,RP,S,CFJ3(40)
     1/CENFOR/G9,NC,C2,C11,C4,C5,AEX,MIU,C6,C7,C8,C9,C10,C1
     2/CCFPNP/C16,ETAO,C13,TO,E,CKFF,PI,RI,RO,LT
      EXTERNAL JATEST,PHERTZ,SYS2,ACDCF,ADINTR,CFJ,PHITER,CFCPNP,FTC
     1,F2N1,FIN
      REAL INH,INS,NC,LT,LE,N,NJ,MIU
      INTEGER Z,Z1,Z2
      DATA TA,TI,R,RP,LT,LE,ETAO,INH,INS,NC,TO,
     1CO,DELES,E,ALPHA,DH,DS,MIU,AEX,PI,CKFF/100.,190.,.4134,2.0175
     2,.819,.6962,.13185E-3,.6E-3,.85E-3,6000.,150.,1.60
     3,.8E-5,.33E+8,.65E-5,5.5118,2.559065,.05,-.26
     2,3.141592654,.0216/
      Z=IZ
      INIT=0
      E7=.001
      ERR9=.09
      CO=1.6*(E*(.852E-4))**.60
      ITE=5
      INT=4
      G=E*(.852E-4)
      S=R/RP
      G9=S
      RI=(1.-S)*S*RP
      C1=(.255E-4)*S**2*LT*RP**3/(LE*E*RI)
      C11=.715*(ETAO*RP*3.141592654/(E*RI*60.))**.71
      RO=(1.+S)*S*RP
      C2=C11*(RO/RI)**.29
      C3=ETAO*3.141592654*RP/(E*RI*30.*CO)
      C4=CO
      C5=RI
      RJ=.001157*S**4*RP**4*LT
      RJ=1.*RJ
672   C10=1800.*S*RP*LE*E*RI/(3.141592654*RJ)
```

```
      C6=R0
      C7=C6*C11/C2
      C8=PI*RP*ETA0/(30.*E*RI)
      C9=C8*RI/R0
      C13=1.43*(ETA0*PI*RP/(60.*E*RI))**.71
      C16=C13*C10
      DELH=INH*(1.+S)*2.*RP/DH
      DELS=INS*DS/(2.*RP*(1.-S))
      DELT=ALPHA*RP*((1.+S)*(TO-TA)-(1.-S)*(TI-TA))
      DELTA=DELES+DELT-DELS-DELH
      CK=(1.14E+7)*LE**(8./9.)
      Z1=Z+1
      DO 10 J=1,Z1
   10 PPHI(J)=2.*3.1415927*FLOAT(J-1)/FLOAT(Z)
      DELTA=.0
907   WRITE(6,1)
      READ(5,2,ERR=900,END=900) P,NC,N
1     FORMAT(' GIVE EXTERNAL LOAD,CAGE SPEED AND INNER RING SPEED')
2     FORMAT()
      MIC=1
      II=MIC+IFIX(5.*P/NC)
      IF(II .GT. 20)II=20
      INIT=0
      PME=(NC/S+(1.-S)*N/(2.*S))/2.
      PMA=(-NC/S+(1.-S)*N/(2.*S))/2.1
      DO 1050 J=1,IZ3
      CFJ3(J)=0.
      SCF(J)=0.
1050  CF(J)=0.
      PC=(.225E-4)*S**2*RP**3*LT*NC**2
      DELPC=(PC/CK)**.9
      DELTA1=DELTA+2.*DELPC
      FAC=3.5
      IF(P .GT. 1500.)FAC=3.6
      IF(P .LT. 320.)FAC=2.
      DO 19 J=1,Z
      DL=((FAC*(P+CF(J))/(CK*FLOAT(Z)))**.9)*(ABS(COS(PPHI(J))))**1.5
      IF(J .EQ. 1)ARRR=DL
      IF(DL .LE. .0) DL=.0
      IF(DELTA1) 17,17,18
17    DDEL(J)=DL-DELTA1*ABS(COS(.5*PPHI(J)))/5.
      GO TO 19
   18 DIN=DELTA1*COS(PPHI(J))/5.
      IF(DIN .LT. 0.) DIN=.0
      DDEL(J)=DL+DIN
   19 CONTINUE
      DDEL(Z1)=DDEL(1)+ARRR+DELTA/2.+DELPC
      IT=50
      L2=0
      NSZ=Z
      NIZ=IFIX(ACOS(DELTA1/(2.*DDEL(Z1)))/PPHI(2))
      L1=10
      NZ=2*NIZ+1
      IF(NZ+1-IZ1)200,201,200
200   PRINT 202,NZ
202   FORMAT(' NUMBER OF ROLLERS IN CONTACT IS NOT IZ1, NZ=',I2)
      GO TO 900
201   DO 877 J=1,NIZ
      DEL(J)=DDEL(J)
  877 DEL(NZ-J+1)=DDEL(J+1)
      DEL(NZ+1)=DDEL(Z+1)
      CFJ3(1)=CF(1)
```

```
      DEL(NIZ+1)=DDEL(NIZ+1)
      PHI(1)=PPHI(1)
  601 DO 991 J=1,NIZ
      PHI(J+1)=PPHI(J+1)
      CFJ3(J+1)=CF(J+1)
      CFJ3(NZ-J+1)=CFJ3(J+1)
  991 PHI(NZ-J+1)=PHI(J+1)
      PHI(NZ+1)=PHI(1)
      CFJ3(NZ+1)=CFJ3(1)
      L2=L2+1
      ANG=PPHI(2)
      Z=NZ
      Z1=Z+1
      DO 85 K=1,Z
   85 PD(K,1)=PC/CK
      PD(Z+1,1)=-P/CK
      DO 20 K=1,Z
      DO 30 J=2,Z1
      IF((J-1) .EQ. K) PD(K,J)=1.
      IF((J-1) .NE. K) PD(K,J)=.0
   30 CONTINUE
   20 PD(K,Z+2)=-1.
      DO 40 J=2,Z1
   42 PD(Z+1,J)=COS(PHI(J-1))
   40 CONTINUE
      PD(Z+1,Z+2)=1.0
      Z2=Z+2
      EPS1=(.1E-35)
      IERR1=0
      IERR2=0
      EPS3=.0001*DEL(1)
      IRR=0
      CALL NSIMEQ(JATEST,PD,QD,A,Z1,IR,JC,Z1,Z2,
     10,25,SI,DS,SF,EPS1,EPS2,EPS3,Q,DEL,DELNEW,DELX,IERR1,IERR2)
      IF(IERR1 .EQ. -1) IRR=3
      IF(IERR2 .EQ. -1) IRR=2
      IF(IRR-2) 903,902,901
  903 NIZN=IFIX(ACOS(DELTA1/(2.*DELNEW(Z1)))/PHI(2))
      IF(NIZN-NIZ) 909,908,909
  909 WRITE(6,119) NIZN
  119 FORMAT(' THE NUMBER OF ROLLERS IN CONTACT IS 2*NIZ+1 ,NIZ =',I3)
     GO TO 900
  908 Z=NSZ
      DDEL(1)=DELNEW(1)
      NIZ2=NIZ+1
      DO 111 J=1,NIZ
      DDEL(J+1)=DELNEW(J+1)
  111 DDEL(Z-J+1)=DELNEW(J+1)
      DO 600 J=1,NIZ2
      BETA=ASIN(DELNEW(Z1)*SIN(PHI(J))/(RP*(1.+S)))
      DDELO(J)=DELNEW(Z1)*COS(PHI(J))-DELTA/2.-DDEL(J)
     1+RP*(1.+S)*(1.-COS(BETA))
  600 IF(DDELO(J) .LT. 0.) DDELO(J)=0.
   50 DO 988 J=1,NIZ
  988 DDELO(Z-J+1)=DDELO(J+1)
      DDEL(Z+1)=DELNEW(Z1)
      NIZ1=NIZ+2
      NIZ2=Z-NIZ
      DO 113 J=NIZ1,NIZ2
      DDELO(J)=(PC/CK)**0.9
  113 DDEL(J)=0.
```

```
      DO 117 J=1,Z
  117 PHI(J)=PPHI(J)
      IWRMX=0
      DO 100 J=1,Z
      IF(DDEL(J)) 299,299,300
  300 PL(J)=CK*DDEL(J)**1.11
      GO TO 703
  299 PL(J)=0.
  703 IF(DDELO(J)) 302,302,301
  301 PLO(J)=CK*DDELO(J)**1.11
      GO TO 702
  302 PLO(J)=0.
  702 PVER=PL(J)*COS(PHI(J))
      W(J)=PL(J)/(E*RI*LE)
      IF(W(J) .GT. (.13E-5))IWRMX=1
      WO(J)=PLO(J)/(E*RO*LE)
100   IF(WO(J) .GT. (.13E-5))IWRMX=1
      W(IZ3)=W(1)
      IF(P-300.)704,704,705
705   IWRMX=1
      GO TO 620
704   DO 137 J=1,Z
      IF(W(J)-W(J-1))3,4,3
    4 VI(J)=VI(J-1)
      GO TO 5
    3 IF(W(J)) 38,38,35
   35 PR=(W(J)/(2.*3.141595))**.5
      BB=4.*PR*RI
      B(1)=-4.*PR
      B(2)=-B(1)
      FK=1.
      RR=RI
      VI(J)=SIM3NI(PHERTZ,B,.01,.TRUE.,100,1.,$39)
      GO TO 5
   39 PRINT 8,J
    8 FORMAT(' INTEGRAL DOES NOT CONVER.,J=',I4)
      GO TO 5
   38 VI(J)=0.
    5 IF(WO(J)-WO(J-1))6,7,6
    7 VIO(J)=VIO(J-1)
      GO TO 137
    6 IF(WO(J)) 136,136,135
  135 PR=(WO(J)/(2.*3.141595))**.5
      GPR=G*PR
      B(1)=-4.*PR
      RR=RO
      B(2)=-B(1)
      BB=B(1)*RO
      VIO(J)=SIM3NI(PHERTZ,BB,.01,.TRUE.,100,1.,$130)
      GO TO 137
  130 PRINT 140,J
  140 FORMAT(' VIO DOES NOT CONVERGE,J=',I3)
      GO TO 137
  136 VIO(J)=0.
  137 CONTINUE
      WO(IZ3)=WO(1)
      W(IZ3)=W(1)
      VI(IZ3)=VI(1)
      VIO(IZ3)=VIO(1)
      GO TO 620
  901 WRITE (6,9)
      GO TO 900
  902 WRITE(6,11
```

```
 31  FORMAT()
  9  FORMAT('PROGRAM ENDED BY ERROR IERR1=-1')
 11  FORMAT('PROGRAM ENDED BECAUSE IERR2=-1')
     GO TO 900
620  IF(IN1T)602,603,602
603  DO 604 KA=1,IZ3
     NJ(KA)=PME+PMA*(COS(PPHI(KA)))/S
     IF((NJ(KA)-NC/S) .LT. 0.)NJ(KA)=NC/S
     SLY=S*NJ(KA)-NC
     IF(IWRMX-1)640,641,640
641  CALL CFCPNP(CF(KA),ACC,DCF,N,NJ(KA),W(KA))
     GO TO 639
640  CALL ACDCF(ACC,DCF,N,NJ(KA),VI(KA),VIO(KA),W(KA),CF(KA))
639  CF(KA)=CF(KA)*LE*E*RI*ABS(NJ(KA))*MIU/NJ(KA)
     IF(CF(KA)-(.1E+5))604,642,642
642  PRINT 31,CF(KA)
     GO TO 900
604  CONTINUE
602  DO 618 LI=1,IZ3
     IF(MIU .LT. (.1E-10))GO TO 618
     CF(LI)=CF(LI)/(LE*E*RI)*ABS(NJ(LI))/(NJ(LI)*MIU)
618  SCF(LI)=CF(LI)
     IF(IN1T)1020,1020,673
1020 I3=IZ*II+1
     DO 680 J=1,IZ3
680  X(J)=FLOAT(J-1)/FLOAT(IZ)
     DX=1./FLOAT(IZ*II)
     XB=0.
     DO 681 J=1,I3
     IF(J .GT. (3*II) .AND. J .LT. (I3-3*II))GO TO 682
     CNJ(J)=GNINT(X,NJ,IZ3,3,XB,D,$683)
     CVI(J)=GNINT(X,VI,IZ3,3,XB,D,$683)
     CVIO(J)=GNINT(X,VIO,IZ3,3,XB,D,$683)
     CW(J)=GNINT(X,W,IZ3,2,XB,D,$683)
     IF(CW(J) .LT. 0.)CW(J)=0.
     CCF(J)=GNINT(X,CF,IZ3,3,XB,D,$683)
     GO TO 681
682  CNJ(J)=STINT(X,NJ,IZ3,3,XB,D,$683)
     CVI(J)=STINT(X,VI,IZ3,3,XB,D,$683)
     CVIO(J)=STINT(X,VIO,IZ3,3,XB,D,$683)
     CW(J)=STINT(X,W,IZ3,2,XB,D,$683)
     IF(CW(J) .LT. 0.)CW(J)=0.
     CCF(J)=STINT(X,CF,IZ3,3,XB,D,$683)
681  XB=XB+DX
673  CALL SYS2(N,CNJ,CVI,CVIO,CW,CCF,IZ,I3,ITE,E7,IER,IWRMX,II,CAF0)
     GO TO 685
683  PRINT 686
686  FORMAT(' CONTINGENCY IN INTERPOLATING SUBROUTINE')
     GO TO 900
685  DO 687 J=1,I3,II
     KK=J/II+1
     NJ(KK)=CNJ(J)
687  CF(KK)=CCF(J)
     IF(IER+1) 605,606,605
606  PRINT 607,IER
607  FORMAT(' DIVERGING SYS2,IER=',I3)
     GO TO 900
605  IF(MIU) 608,614,608
608  ER8=0.
     ER9=0.
     DO 609 K=1,IZ3
```

```
      IF(SCF(K)) 610,611,610
 610  ER8=(CF(K)-SCF(K))/SCF(K)
      GO TO 609
 611  IF(CF(K))613,609,613
 613  ER8=(CF(K)-SCF(K))/CF(K)
 609  IF(ABS(ER8) .GT. ER9)ER9=ABS(ER8)
      IF(ER9-ERR9)614,614,615
 615  DO 616 K=1,IZ3
 616  CF(K)=CF(K)*LE*E*RI*(ABS(NJ(K))/NJ(K))*MIU
      INIT=INIT+1
      IF(INIT-INT)601,601,621
 621  PRINT 622,ER9
 622  FORMAT(' GENERAL ITERATION DIVERGING, ER9=',())
 614  EPNJ=(1.-S)*N/(2.*S)
      TTF=0.
      ADVI=ETA0*3.14159265*RP/(30.*E*RI)
      DO 617 J=1,IZ3
      SS5=1.
      SS6=1.
      V=ADVI*(N*(1.-S)-NC-S*NJ(J))
      U=ADVI*(N*(1.-S)+NC+S*NJ(J))/2.
      IF(W(J) .LT. 0.)SS5=-1.
      IF(W(J)) 627,626,627
 627  WA1=ABS(W(J))
      IF(ABS(W(J)) .LT. (.1E-37))WA1=.1E-36
      WA2=SS5*WA1**AEX
      GO TO 628
 626  WA2=0.
 628  IF(U .LT. 0.)SS6=-1.
      IF(U)629,630,629
 629  WA3=ABS(U)
      IF(ABS(U) .LT. (.1E-37))WA3=.1E-36
      WA4=SS6*WA3**.7
      WA5=SS6*WA3**.71
      GO TO 631
 630  WA5=0.
      WA4=0.
 631  U1=N*(1.-S)
      U2=NC+S*NJ(J)
      RF=PHITER(U1,U2,W(J))
      H(J)=CO*WA4*WA2*RF
      IF(H(J))633,634,633
 634  TF=0.
      GO TO 632
 633  IF(IWRMX-1)637,638,637
 638  TFF=FTC(U1,U2,W(J))
      TF=TFF*W(J)
      GO TO 632
 637  TF=-1.43*WA5/2.+VI(J)*V/H(J)
 632  TTF=TTF+TF
      SLIP(J)=(NJ(J)-EPNJ)*100./EPNJ
      IF(W(J))635,636,635
 636  TC(J)=0.
      GO TO 617
 635  TC(J)=TF/W(J)
 617  CONTINUE
      TTC=TTF*E*LE*RI/P
      EPNC=(1.-S)*N/2.
      CSLIP=(NC-EPNC)*100./EPNC
      EPNJC=NC/S
      PRINT 932,P,N,EPNJ,CSLIP,EPNJC,EPNC,TTC,CAFO
      PRINT 731
 731  FORMAT(/,' J      DEF        W          TC          NJ          SLIP
     1RCF        H',/)
```

```
932   FORMAT(/,'  P ='，E10.5,'  N ='，E10.5,'  EPNJ ='，E10.5,'  CSLIP ='
     1E10.5,/,/,,'EPNJC =',E10.5,'  EPNC =',E10.5,'  TTC =',E10.5,'  CF
     2=',E10.5,/,/)
      DO 371 K=1,IZ3
      J=K
371   PRINT 331,J,DDEL(J),W(J),TC(J),NJ(J),SLIP(J),CF(J),H(J)
331   FORMAT(1X,I2,2E9.4,5(1X,E9.4))
      GO TO 907
  900 END
```

```
      FUNCTION PHERTZ(X,FK)
C         THIS SUBROUTINE PERFORMS THE INTEGRATION REQUIRED
C         WHEN THE HIGGINSON'S METHOD TO CALCULATE THE TRACTION FORCES
C         IS USED.
      COMMON /HERTZ/G,PR,RR,BB
      S=1.
      A=1.-(RR*X/BB)**2
      IF(A)1,2,3
1     S=-1.
3     AR=S*G*PR*((ABS(A))**.5)
      IF(AR)4,2,5
4     IF(AR+87.)6,7,7
6     PHERTZ=0.
      RETURN
5     IF(AR-87)7,7,8
8     PRINT 9,AR
9     FORMAT(' USE DOUBLE PRECISION,AR='())
7     PHERTZ=EXP(AR)
      RETURN
2     PHERTZ=1.
      RETURN
      END
```

```
      FUNCTION PHITER(U1,U2,W)
C     SUBPROGRAM TO GET FILM THICKNESS THERMAL REDUCTION FACTOR
      COMMON /JAT/ANG,PC,CK,P,DELTA,RP,S,CFJ3(40)
     1/CENFOR/AFARTA,NC,C2,C11,C4,C5,AEX,MIU,C6,C7,C8,C9,C10,C1/
     1CCEPNP/C16,ETA0,C13,TO,E,CKFF,PI,RI,RO,TL
      Q=ETA0*((RP*PI/30.)**2)*(U1+U2)**2/(2.*CKFF*TO)
      IF(W)2,2,1
2     X=0.
      GO TO 3
1     X=E*((W/(2.*PI))**.5)/110000.
3     PHITER=EXP(-(.5677+.0348*X)*EXP((.4003+.0311*X)*ALOG(Q)))
      IF(PHITER .LT. .1)PHITER=.1
      RETURN
      END
```

```
      SUBROUTINE JATEST(Q,Z1,DEL,Z3,Z2,A)
C     SUBROUTINE TO BE USED WITH NSIMEQ (MATH-PACK) TO
C     SOLVE THE FIRST SET OF EQUATIONS:LOAD DISTRIBUTION.
      COMMON /JAT/ANG,PC,CK,P,DELTA,RP,S,CFJ3(40)
      DIMENSION A(Z1,Z2),DEL(Z1),Q(Z1,Z2)
      INTEGER Z,Z3
      Z=Z1-1
      A(Z1,Z1)=0.
      A(Z1,Z2)=P/CK
      DO 820 K=1,Z
      ZA=Z/2+1
      IF(K-ZA) 1,1,2
1     PHI1=ANG*FLOAT(K-1)
      GO TO 4
2     PHI1=ANG*FLOAT(Z1-K)
4     IF(ABS(DEL(Z1)*SIN(PHI1)/(RP*(1.+S)))-1.) 11,11,10
10    PRINT 5,DEL(Z1)
5     FORMAT(' DIVERGING,DEL(Z1)='())
11    BETA=ASIN(DEL(Z1)*SIN(PHI1)/(RP*(1.+S)))
852   DELO=DEL(Z1)*COS(PHI1)-DELTA/2.-DEL(K)
     1+RP*(1.+S)*(1.-COS(BETA))
      IF(DEL(K) .LT. 0.)SI=-1.
      IF(DELO .LT. 0.)SSI=-1.
      IF(DEL(K) .GT. 0.)SI=1.
      IF(DELO .GT. 0.)SSI=1.
      ATI=ABS(DEL(K))
      ATO=ABS(DELO)
      IF(ATI)110,111,110
111   A11=.1E-11
      A33=0.
      GO TO 114
110   A11=SI*ATI**.11
      A33=SI*ATI**1.11
114   IF(ATO)116,117,116
117   A22=.1E-11
      A44=0.
      GO TO 747
116   A22=SSI*ATO**.11
      A44=SSI*ATO**1.11
747   A55=A22+A11
      A(Z1,K)=1.11*A11*COS(PHI1)
      A(Z1,Z+2)=A(Z1,Z+2)-A33*COS(PHI1)
      A(K,K)=1.11*(A11+A22)
      IF(COS(BETA)-1.)100,99,100
99    A(K,Z1)=-1.11*A22*COS(PHI1)
      GO TO 101
100   A(K,Z1)=-1.11*A22*(COS(PHI1)+SIN(PHI1)*SIN(BETA)
     1/((1.-(COS(BETA))**2)**.5))
101   A(K,Z+2)=(CFJ3(K)-PC)/CK-A33+A44
752   A(Z1,Z1)=A(Z1,Z1)-A(K,Z1)*A11*COS(PHI1)/A55
810   DO 820 J=1,Z
      IF(K .NE. J) A(K,J)=0.
820   CONTINUE
      RETURN
      END
55:>
```

```
      SUBROUTINE CFCPNP(NCF,ACC,DCF,N,NJ,W)
C     SUBROUTINE TO EVALUATE THE CAGE FORCE,NCF, THE ROLLER SPINNING
C     ACCELERATION,ACC,AND THE DERIVATIVE OF NCF WITH RESPECT TO
C     THE INNER RING SPEED.
      COMMON /CENFOR/S,NC,C2,C11,C4,C5,AEX,MIU,C6,C7,C8,C9,C10,C1
     1/CCFPNP/C16,ETA0,C13,T0,E,CKFF,PI,RI,RO,DUMM
      REAL NCF,NC,MIU,NJ,N
      INDEX=0
      SSS=1.
      U10=0.
      U20=-NC+S*NJ
      IF(NJ)5,4,5
4     W0=(RI/RO)*(W+C1*NC**2)
      GO TO 6
5     W0=(RI/RO)*(W+C1*NC**2+MIU*NJ*NCF/ABS(NJ))
      IF(W0)12,12,6
12    TC0=0.
      GO TO 13
6     TC0=FTC(U10,U20,W0)
13    IF(W)14,14,9
14    TCI=0.
      TCI1=0.
      DTCIN=0.
      GO TO 15
9     U1I=N*(1.-S)
      U2I=S*NJ+NC
      TCI=FTC(U1I,U2I,W)
      IF(INDEX-1)8,10,11
8     TCI1=TCI
      AU1I=U1I
      ATN=N
      DELTUI=.00001*N
      INDEX=INDEX+1
      N=N+DELTUI
      GO TO 9
10    AAA=TCI
      N=N-2.*DELTUI
      INDEX=INDEX+1
      GO TO 9
11    DTCIN=(AAA-TCI)/(2.*DELTUI)
      N=ATN
      U1I=AU1I
      TCI=TCI1
15    NCF=TC0*(RO/RI)*W0-TCI*W
      DCF=-W*DTCIN
      IF(NJ .LT. 0.)SSS=-1.
      IF(ABS(NJ) .LT. (.1E-35))NJ=SSS*(.1E-35)
      ACC=C10*(TCI*W+(RO/RI)*TC0*W0-MIU*NJ*ABS(NCF)/ABS(NJ))/NC
31        FORMAT(' CFCP,'())
          RETURN
          END
51:>
```

```
      SUBRØUTINE SYS2(N,NJ,VI,VIØ,W,CF,IZ,IZ3,ITE,E7,IER,IWRMX,II,A13)
C          SUBRØUTINE TØ  BE USED WHEN GIVEN A CAGE SPEED AND INNER
C          RING SPEED,THE TOTAL CAGE FØRCE AND OTHER OPERAYING VARIABLES
C          ARE LOOKED FOR.IT MUST BE USED IF FOR A SPECIFIED CAGE SPEED
C          WE ARE LOOKING FOR THE CAGE FØRCE AS FUNCTIØN ØF THE
C          INNER RING SPEED.
C                                       BY  MANUEL A. MØLINA C.
      CØMMØN /JAT/ANG,PC,CK,P,DELTA,RP,S,CFJ3(40)
     1/CENFØR/DUMMM,NC,C2,C11,C4,C5,AEX,MIU,C6,C7,C8,C9,C10,C1
     1/CCFPNP/C16,ETA0,C13,TØ,E,CKFF,PI,RI,RØ,TL
      DIMENSIØN NJ(241),VI(241),VIØ(241),W(241),CF(241),NPJ(246)
     1,TN(241)
      REAL N,NJ,NC,MIU,NPJ
      H=1./FLØAT(IZ3-1)
      IMI=0
      IM=IZ3+3
      IS2=1
      IT=0
      EPL=.85*NC/S
      EPH=1.3*(1.-S)*N/(2.*S)
      LIM=IZ3-1
      FAC=ABS(20.*((1.-S)*N/2.-NC)/NC)
      IF(FAC .LT. 1.)FAC=1.
      IF(FAC .GT. 2.5)FAC=2.5
29    ITN=0
431    FØRMAT(14E8.3)
26    I=0
      AN=N
      IC=0
      A11=0.
      CLA=NJ(IZ3)-NJ(1)
      TCF=0.
54    CACA=NJ(IZ3)
      DØ 20 KJ=1,IM
      J=KJ-3
      IF(J .EQ. 1)GØ TØ 20
      IF(J .GT. 1)J=J-1
      IF(KJ .LT. 4)J=IZ3-4+KJ
      IF(KJ .LT. 4)NJ(J)=NJ(J)-CLA
99    IF(IWRMX-1)21,22,21
22        CALL CFCPNP(CF(J),ACC,DCF,N,NJ(J),W(J))
92        G0 TØ 23
21        CALL ACDCF(ACC,DCF,N,NJ(J),VI(J),VIØ(J),W(J),CF(J))
23        NPJ(KJ)=ACC
      IF(KJ .LT. 4)NJ(J)=NJ(J)+CLA
      TN(J)=DCF
      IF(KJ-4)20,20,51
51    NJ(J+1)=NJ(J)+H*(251.*NPJ(KJ)+646.*NPJ(KJ-1)-264.*NPJ(KJ-2)
     1+106.*NPJ(KJ-3)-19.*NPJ(KJ-4))/720.
      IF(NJ(J+1) .GT. EPH)NJ(J+1)=EPH
      IF(NJ(J+1) .LT. EPL)NJ(J+1)=EPL
20        CONTINUE
52    IF(ABS((CACA-NJ(IZ3))/NJ(IZ3))-E7)58,58,55
55    CLA=NJ(IZ3)-NJ(1)
      IC=IC+1
      IF(IC-ITE)54,54,11
58    DØ 80 J=1,LIM,II
80    TCF=TCF+CF(J)
      A13=-TCF
      A23=NJ(IZ3)-NJ(1)
      DELN1=-A23
      D=DELN1
      IF(ABS(D/NJ(IZ3)) .LT. E7)G3 , 1,10
```

```
      IF(ABS(DELN1) .GT. 4000.)DELN1=DELN1/10.
      IF(ABS(DELN1) .GT. 1000.)DELN1=DELN1/2.5
      IF(ABS(DELN1) .GT. 1000.)DELN1=DELN1*FAC/2.5
      IF(ABS(DELN1) .GT. 1000.)DELN1=DELN1*FAC/2.5
      IF(ABS(DELN1) .GT. 700.)DELN1=DELN1*FAC/2.5
      NJ(1)=NJ(1)-DELN1
      IF(NJ(1) .LT. (.8*NC/S))NJ(1)=.93*NC/S
      IF(NJ(1) .GT. 1.3*N*(1.-S)/(2.*S))NJ(1)=1.3*N*(1.-S)/(2.*S)
      IT=IT+1
      IF(IT .LT. ITE)GO TO 29
   31 FORMAT ()
 11       IER=-1
          RETURN
 10       IER=IT
          RETURN
          END
```

```
      FUNCTION FTC(U1,U2,W)
C         THIS FUNCTION GIVES THE TRACTION COEFICIENT
      COMMON /JAT/ANG,PC,CK,P,DELTA,RP,S,CFJ3(40)
     1/CENFOR/DUM(14)
     1/CCFPNP/C16,ETA0,C13,T0,E,CKFF,PI,RI,RO,DSAGFJ
C     SUBPROGRAM TO CALCULATE TRACTION COEFICIENTS AT HIGH PRESSURES
      SS=1.
      PV=-(1.-S)*DUM(2)*2.
      IF(ABS(U1) .LT. (.1E-26))PV=2.*DUM(2)*(1.+S)
      I=0
      AL=2.*(U1-U2)/(ABS(U1+U2+PV))
      IF(AL .LT. 0.)SS=-1.
      DV=AL
      IF(ABS(AL)-.2)4,4,5
5     ALA=ABS(AL)
      AL=.14
      I=I+1
4     IF(W)2,2,1
2     X=0.
      GO TO 3
1     X=E*((W/(2.*PI))**.5)/110000.
3     C=-.0143*X-.0357
      D=.046-.031*EXP(-.31136*X**3.41435)
      A=147.507*ATAN(4.76979*(X-1.0597))+.1*(X-1.0597)+210.
      FTC=SS*(C*ABS(AL)+D*(1.-EXP(-A*ABS(AL))))
      IF(I .EQ. 1)FTC=FTC*EXP(.5*ALOG(3)*(.14-ABS(ALA)))
      RETURN
      END
```

```
      SUBROUTINE SYS2(N,NJ,VI,VIO,W,CF,IZ,IZ3,ITE,E7,IER,IWRMX,II)
C     SUBROUTINE FOR THE SECOND SYSTEM OF EQUATIONS IN THE
C           DYNAMIC ANALYSIS OF ROLLING BEARINGS
C         THIS SUBROUTINE MUST BE USED WHEN GIVEN A CAGE SPEED AND
C         A GUESSED SHAFT SPEED,THE EQUILIBRIUM OPERATING CONDITIONS
C         ARE DESIRED.HERE THE RUMBARGER'S MODEL TO CALCULATE
C         THE DRAG FORCE IS USED.
C                                    BY  MANUEL A. MOLINA C.
      COMMON /JAT/ANG,PC,CK,P,DELTA,RP,S,CFJ3(40)
     1/CENFOR/DUMMM,NC,C2,C11,C4,C5,AEX,MIU,C6,C7,C8,C9,C10,C1
     1/CCFPNP/C16,ETA0,C13,T0,E,CKFF,PI,RI,RO,TL
      DIMENSION NJ(241),VI(241),VI0(241),W(241),CF(241),NPJ(246)
     1,TN(241)
      REAL N,NJ,NC,MIU,NPJ
      H=1./FLOAT(IZ3-1)
      RE=4.593*NC*RP**2
      CN=3.87/(RE**.5)
      IF(RE .GT. 300000.)CN=.146/(RE**.2)
      DF=(.1239E-6)*NC**2*RP**4.*CN
      REI=4.593*RP*(1.-S)*NC/100.
      FI=16./REI
      IF(REI .GT. 2500.)FI=(48./REI)*(REI/2500.)**.85596
      RE0=.04593*RP*(1.+S)*NC
      F0=16./RE0
      IF(F0 .GT. 2500.)F0=(48./RE0)*(RE0/2500.)**.85596
      DAI=(.7786E-5)*TL*FI*RP**3.*(1.-S)**3.*NC**2
      DA0=(.7786E-5)*TL*F0*RP**3.*(1.-S)**3.*NC**2
      DRAG=DA0+DAI+DF
      PRINT 31,DA0,DAI,DF,DRAG,E,IZ3
      IF(IZ3 .GT. 241)GO TO 11
      DIV=100000.
      CA=0.
      IMI=0
      IM=IZ3+3
      IS2=1
      IT=0
      LIM=IZ3-1
29    ITN=0
      SSS=1.
431    FORMAT(14E8.3)
26    I=0
      AN=N
      IC=0
      A11=0.
      CLA=NJ(IZ3)-NJ(1)
      TCF=0.
54    CACA=NJ(IZ3)
      DO 20 KJ=1,IM
      J=KJ-3
      IF(J .EQ. 1)GO TO 20
      IF(J .GT. 1)J=J-1
      IF(KJ .LT. 4)J=IZ3-4+KJ
      IF(KJ .LT. 4)NJ(J)=NJ(J)-CLA
99    IF(IWRMX-1)21,22,21
22        CALL CFCPNP(CF(J),ACC,DCF,N,NJ(J),W(J))
92        GO TO 23
21        CALL ACDCF(ACC,DCF,N,NJ(J),VI(J),VI0(J),W(J),CF(J))
23       NPJ(KJ)=ACC
      IF(KJ .LT. 4)NJ(J)=NJ(J)+CLA
      TN(J)=DCF
      IF(KJ-4)20,20,51
```

```
  1
        NJ(J+1)=NJ(J)+H+(251.*NPJ(KJ)+646.*NPJ(KJ-1)-264.*NPJ(KJ-2)
       1+106.*NPJ(KJ-3)-19.*NPJ(KJ-4))/720.
 20         CONTINUE
 52     IF(ABS((CACA-NJ(IZ3))/NJ(IZ3))-E7)58,58,55
 55     CLA=NJ(IZ3)-NJ(1)
        IC=IC+1
        IF(IC-ITE)54,54,11
 58     DO 80 J=1,LIM,II
        A11=A11+TN(J)
 80     TCF=TCF+CF(J)
            SS=1.
 25     A13=TCF+DRAG/(E*.8*TL*RI)
        A23=NJ(IZ3)-NJ(1)
        PRINT 31,A11,A13,A23,N,NJ(1)
 31     FORMAT()
        IF(IS2 .EQ. 1)GO TO 19
 2      IF(A11 .LT. 0.)SS=-1.
        IF(ABS(A11) .LT. (.1E-10))A11=SS*(.1E-10)
        DELTAN=A13/A11
        IF(ABS(DELTAN/AN) .LT. (E7))GO TO 19
        IF(ABS(DELTAN) .GT. 4000.)DELTAN=DELTAN/50.
        IF(ABS(DELTAN) .GT. 1000.)DELTAN=DELTAN/10.
        IF(ABS(DELTAN) .GT. 1000.)DELTAN=DELTAN/10.
        IF(ABS(DELTAN) .GT. 1000.)DELTAN=DELTAN/10.
        IF(ABS(DELTAN) .GT. 700.)DELTAN=DELTAN/10.
        IF(DELTAN .GT. 300.)DELTAN=DELTAN/4.
        DELTAN=DELTAN/2.1
        IF(P .GT. 700.)DELTAN=DELTAN/3.5
        N=AN-DELTAN
        IF(N .LT. (1.6*NC/(1.-S)))N=2.4*NC/(1.-S)
        IS2=1
        ITN=ITN+1
        IF(ITN-ITE)26,26,11
 19     DELN1=-A23
        D=DELN1
        IF(ABS(D/NJ(IZ3)) .LT. E7)GO TO 33
        IF(ABS(DELN1) .GT. 4000.)DELN1=DELN1/10.
        IF(ABS(DELN1) .GT. 1000.)DELN1=DELN1/2.5
        IF(ABS(DELN1) .GT. 1000.)DELN1=DELN1/2.5
        IF(ABS(DELN1) .GT. 1000.)DELN1=DELN1/2.5
        IF(ABS(DELN1) .GT. 700.)DELN1=DELN1/2.5
        NJ(1)=NJ(1)-DELN1
        IF(NJ(1) .LT. (.8*NC/S))NJ(1)=.93*NC/S
        IF(NJ(1) .GT. 1.3*N*(1.-S)/(2.*S))NJ(1)=1.3*N*(1.-S)/(2.*S)
        IT=IT+1
        IF(IT-ITE)29,29,11
 33     IF(ABS(A13/A11)/AN .LT. E7 .AND. ABS(D/NJ(IZ3)) .LT. E7)GO TO 10
        IS2=0
        GO TO 2
 11         IER=-1
            RETURN
 10         IER=IT
            RETURN
            END
SCAN:0
EOF:115
0:>
```

```
      SUBROUTINE ACDCF(ACC,DCF,N,NJ,VI,VI0,W,CF)
      COMMON/CENFOR/S,NC,C2,C11,C4,RI,AEX,MIU,R0,C7,C8,C9,C10,C1
C     SUBROUTINE TO CALCULATE THE DERIVATIVE OF NJ,ACC,AND THE
C     DERIVATIVE OF THE CAGE FORCE RESPECT TO INNER RING SPEED N,
C     WHEN THE HIGGINSON'S METHOD IS USED TO CALCULATE THE TRACTION
C     FORCES.
      REAL N,NC,NJ,MIU
      IN=0
22    A4=MIU*ABS(NJ)*CF/NJ
23    A3=W+C1*N**2+A4
      IF(A3 .LT. 0.)SS3=-1.
      U10=0.
      U20=S*NJ-NC
      IF(A3)33,33,99
33    FTCF0=1.
      GO TO 29
99    FTCF0=PHITER(U10,U20,A3)
29    SS1=1.
      SS2=1.
      SS3=1.
      SS4=1.
      SS5=1.
      U1I=(1.-S)*N
      U2I=NC+S*NJ
32    IF(W)35,35,30
35    FTCFI=1.
      DCF=0.
      FI=0.
      GO TO 82
30    FTCFI=PHITER(U1I,U2I,W)
34    UE=(U10+U20)/2.
      VE=(U10-U20)
      IF(UE .LT. 0.)SS1=-1.
      IF(VE .LT. 0.)SS2=-1.
      DUE=C9*ABS(UE)
      CVE=C9*ABS(VE)
      HE=C4*FTCF0*DUE**.7*(ABS(A3))**AEX
      FEX=.715*SS1*DUE**.71+VI0*CVE/HE
      UI=(U1I+U2I)/2.
      V=U1I-U2I
      IF(W)80,80,81
80    FI=0.
      DCF=0.
      GO TO 82
81    IF(UI .LT. 0.)SS4=-1.
      IF(V .LT. 0.)SS5=-1.
      DUI=C8*ABS(UI)
      DV=C8*ABS(V)
      HI=C4*FTCFI*DUI**.71*W**AEX
      FI=.715*SS4*DUI**.71+VI*DV/HI
      IF(IN-1)83,84,85
83    SFI=FI
      SHI=HI
      SFEX=FEX
      SHE=HE
      IN=IN+1
      SDV=DV
      SDUI=DUI
      N=1.0001*N
      GO TO 29
84    FIS=FI
      IN=IN+1
      N=.9998*N
```

```
      GO TO 29
85    DCF=(FIS-FI)/(.0002*N)
      DUI=SDUI
      FI=SFI
      DV=SDV
      HE=SHE
      FEX=SFEX
      HI=SHI
82    CF=FEX*R0/RI-FI
      ACC=(C10*(FI+R0*FEX/RI)-MIU*ABS(NJ)*ABS(CF)/NJ)/NC
      RETURN
31        FØRMAT()
          END
76:>
```

APPENDIX B

RESULTS

The tabulated results obtained are plotted and presented in Figures 5 to 29. A few of the tables obtained are included, Tables 2 to 13, as example of the results given by the computations.

Table 2. Cage Force. Applied Load 2400 lb_f. Cage Speed 400 rpm

NC	P	EPNC	CSLIP	N	CAGE FØRCE
.400000+03	.240000+04	.400800+03	-.199601-02	.100818+04	.496097-05
.400000+03	.240000+04	.401600+03	-.398405-02	.101020+04	.827517-05
.400000+03	.240000+04	.402400+03	-.596420-02	.101221+04	.106011-04
.400000+03	.240000+04	.403200+03	-.793648-02	.101422+04	.122452-04
.400000+03	.240000+04	.404000+03	-.990096-02	.101623+04	.134283-04
.400000+03	.240000+04	.404800+03	-.118577-01	.101825+04	.142936-04
.400000+03	.240000+04	.405600+03	-.138067-01	.102026+04	.149358-04
.400000+03	.240000+04	.406400+03	-.157480-01	.102227+04	.154182-04
.400000+03	.240000+04	.407200+03	-.176817-01	.102428+04	.157839-04
.400000+03	.240000+04	.408000+03	-.196078-01	.102630+04	.160631-04
.400000+03	.240000+04	.408800+03	-.215263-01	.102831+04	.162767-04
.400000+03	.240000+04	.409600+03	-.234374-01	.103032+04	.164402-04
.400000+03	.240000+04	.410400+03	-.253410-01	.103233+04	.165644-04
.400000+03	.240000+04	.411200+03	-.272372-01	.103434+04	.166578-04
.400000+03	.240000+04	.412000+03	-.291261-01	.103636+04	.167264-04
.400000+03	.240000+04	.416800+03	-.403069-01	.104843+04	.168257-04
.400000+03	.240000+04	.421600+03	-.512332-01	.106050+04	.166680-04
.400000+03	.240000+04	.426400+03	-.619135-01	.107258+04	.164134-04
.400000+03	.240000+04	.431200+03	-.723560-01	.108465+04	.161184-04
.400000+03	.240000+04	.444000+03	-.990989-01	.111685+04	.152159-04
.400000+03	.240000+04	.456800+03	-.124343+00	.114905+04	.136859-04
.400000+03	.240000+04	.469600+03	-.148211+00	.118125+04	.136746-04
.400000+03	.240000+04	.482400+03	-.170812+00	.121344+04	.124367-04
.400000+03	.240000+04	.495200+03	-.192245+00	.124564+04	.122964-04
.400000+03	.240000+04	.548000+03	-.270073+00	.137845+04	.124259-04

GIVE EXTERNAL LØAD AND CAGE SPEED
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Table 3. Cage Force. Applied Load 200 lb.. Shaft Speed 1750 rpm

NC	P	EPNC	CSLIP	N	CAGE FORCE
.695706+03	.200000+03	.697098+03	-.199600-02	.175350+04	.254789-07
.695706+03	.200000+03	.698489+03	-.398405-02	.175700+04	.497780-07
.695706+03	.200000+03	.699881+03	-.596419-02	.176050+04	.732585-07
.695706+03	.200000+03	.701272+03	-.793648-02	.176400+04	.958461-07
.695706+03	.200000+03	.702663+03	-.990096-02	.176750+04	.117575-06
.695706+03	.200000+03	.704055+03	-.118577-01	.177100+04	.138479-06
.695706+03	.200000+03	.705446+03	-.138067-01	.177450+04	.158589-06
.695706+03	.200000+03	.706838+03	-.157480-01	.177800+04	.177934-06
.695706+03	.200000+03	.708229+03	-.176816-01	.178150+04	.196543-06
.695706+03	.200000+03	.709620+03	-.196078-01	.178500+04	.214443-06
.695706+03	.200000+03	.711012+03	-.215263-01	.178850+04	.231658-06
.695706+03	.200000+03	.712403+03	-.234374-01	.179200+04	.248213-06
.695706+03	.200000+03	.713795+03	-.253410-01	.179550+04	.264130-06
.695706+03	.200000+03	.715186+03	-.272373-01	.179900+04	.279432-06
.695706+03	.200000+03	.716577+03	-.291261-01	.180250+04	.294139-06
.695706+03	.200000+03	.724926+03	-.403070-01	.182350+04	.370959-06
.695706+03	.200000+03	.733274+03	-.512332-01	.184450+04	.430784-06
.695706+03	.200000+03	.741623+03	-.619135-01	.186550+04	.476538-06
.695706+03	.200000+03	.749971+03	-.723560-01	.188650+04	.510383-06
.695706+03	.200000+03	.772234+03	-.990989-01	.194250+04	.548439-06
.695706+03	.200000+03	.794496+03	-.124343+00	.199850+04	.494580-06
.695706+03	.200000+03	.816759+03	-.148211+00	.205450+04	.483296-06
.695706+03	.200000+03	.839022+03	-.170812+00	.211050+04	.495952-06
.695706+03	.200000+03	.861284+03	-.192245+00	.216650+04	.485190-06
.695706+03	.200000+03	.953117+03	-.270073+00	.239750+04	.447549-06

GIVE EXTERNAL LOAD AND CAGE SPEED

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Table 4. Cage Force. Applied Load 800 lb_f. Cage Speed 700 rpm

NC	P	EPNC	CSLIP	N	CAGE FØRCE
.700000+03	.800000+03	.701400+03	-.199600-02	.176432+04	.270184-06
.700000+03	.800000+03	.702800+03	-.398405-02	.176784+04	.515602-06
.700000+03	.800000+03	.704200+03	-.596419-02	.177137+04	.743489-06
.700000+03	.800000+03	.705600+03	-.793648-02	.177489+04	.953583-06
.700000+03	.800000+03	.707000+03	-.990096-02	.177841+04	.114732-05
.700000+03	.800000+03	.708400+03	-.118577-01	.178193+04	.132600-05
.700000+03	.800000+03	.709800+03	-.138067-01	.178545+04	.149082-05
.700000+03	.800000+03	.711200+03	-.157480-01	.178897+04	.164287-05
.700000+03	.800000+03	.712600+03	-.176817-01	.179249+04	.178315-05
.700000+03	.800000+03	.714000+03	-.196078-01	.179602+04	.191256-05
.700000+03	.800000+03	.715400+03	-.215264-01	.179954+04	.203194-05
.700000+03	.800000+03	.716800+03	-.234374-01	.180306+04	.214206-05
.700000+03	.800000+03	.718200+03	-.253411-01	.180658+04	.224361-05
.700000+03	.800000+03	.719600+03	-.272373-01	.181010+04	.233724-05
.700000+03	.800000+03	.721000+03	-.291261-01	.181362+04	.242353-05
.700000+03	.800000+03	.729400+03	-.403070-01	.183475+04	.281475-05
.700000+03	.800000+03	.737800+03	-.512333-01	.185588+04	.304700-05
.700000+03	.800000+03	.746200+03	-.619136-01	.187701+04	.317467-05
.700000+03	.800000+03	.754600+03	-.723561-01	.189814+04	.323266-05
.700000+03	.800000+03	.777000+03	-.990990-01	.195449+04	.282369-05
.700000+03	.800000+03	.799400+03	-.124343+00	.201083+04	.220407-05
.700000+03	.800000+03	.821800+03	-.148211+00	.206718+04	.257444-05
.700000+03	.800000+03	.844200+03	-.170812+00	.212353+04	.251906-05
.700000+03	.800000+03	.866600+03	-.192245+00	.217987+04	.246877-05
.700000+03	.800000+03	.959000+03	-.270073+00	.241230+04	.231367-05

GIVE EXTERNAL LØAD AND CAGE SPEED

Table 5. Cage Force. Applied Load 400 lb. Shaft Speed 3600 rpm

NC	P	EPNC	CSLIP	N	CAGE FORCE
.143117+04	.400000+03	.143403+04	-.199601-02	.360720+04	.307473-07
.143117+04	.400000+03	.143689+04	-.398405-02	.361440+04	.156722-06
.143117+04	.400000+03	.143975+04	-.596420-02	.362160+04	.229318-06
.143117+04	.400000+03	.144262+04	-.793649-02	.362880+04	.298294-06
.143117+04	.400000+03	.144548+04	-.990097-02	.363600+04	.363808-06
.143117+04	.400000+03	.144834+04	-.118577-01	.364320+04	.426016-06
.143117+04	.400000+03	.145120+04	-.138067-01	.365040+04	.485066-06
.143117+04	.400000+03	.145407+04	-.157480-01	.365760+04	.541101-06
.143117+04	.400000+03	.145693+04	-.176817-01	.366480+04	.594261-06
.143117+04	.400000+03	.145979+04	-.196078-01	.367200+04	.644678-06
.143117+04	.400000+03	.146265+04	-.215264-01	.367920+04	.692477-06
.143117+04	.400000+03	.146552+04	-.234374-01	.368640+04	.737775-06
.143117+04	.400000+03	.146838+04	-.253411-01	.369360+04	.780686-06
.143117+04	.400000+03	.147124+04	-.272373-01	.370080+04	.821312-06
.143117+04	.400000+03	.147410+04	-.291262-01	.370800+04	.859753-06
.143117+04	.400000+03	.149128+04	-.403070-01	.375120+04	.104933-05
.143117+04	.400000+03	.150845+04	-.512333-01	.379440+04	.117914-05
.143117+04	.400000+03	.152562+04	-.619136-01	.383760+04	.125857-05
.143117+04	.400000+03	.154280+04	-.723561-01	.388080+04	.129155-05
.143117+04	.400000+03	.158860+04	-.990990-01	.399600+04	.118803-05
.143117+04	.400000+03	.163439+04	-.124343+00	.411120+04	.953327-06
.143117+04	.400000+03	.168019+04	-.148211+00	.422640+04	.110880-05
.143117+04	.400000+03	.172599+04	-.170812+00	.434160+04	.108432-05
.143117+04	.400000+03	.177178+04	-.192245+00	.445680+04	.106139-05
.143117+04	.400000+03	.196070+04	-.270073+00	.493200+04	.980908-06

GIVE EXTERNAL LOAD AND CAGE SPEED
>

Table 6. Cage Force. Applied Load 2000 lb_f. Cage Speed 1000 rpm

NC	P	EPNC	CSLIP	N	CAGE FØRCE
.100000+04	.200000+04	.100200+04	-.199601-02	.252046+04	.302139-05
.100000+04	.200000+04	.100400+04	-.398406-02	.252549+04	.520159-05
.100000+04	.200000+04	.100600+04	-.596422-02	.253052+04	.685621-05
.100000+04	.200000+04	.100800+04	-.793651-02	.253555+04	.811117-05
.100000+04	.200000+04	.101000+04	-.990099-02	.254058+04	.907474-05
.100000+04	.200000+04	.101200+04	-.118577-01	.254561+04	.982303-05
.100000+04	.200000+04	.101400+04	-.138067-01	.255065+04	.104101-04
.100000+04	.200000+04	.101600+04	-.157480-01	.255568+04	.108748-04
.100000+04	.200000+04	.101800+04	-.176817-01	.256071+04	.112454-04
.100000+04	.200000+04	.102000+04	-.196078-01	.256574+04	.115426-04
.100000+04	.200000+04	.102200+04	-.215264-01	.257077+04	.117821-04
.100000+04	.200000+04	.102400+04	-.234375-01	.257580+04	.119755-04
.100000+04	.200000+04	.102600+04	-.253411-01	.258083+04	.121317-04
.100000+04	.200000+04	.102800+04	-.272373-01	.258586+04	.122576-04
.100000+04	.200000+04	.103000+04	-.291262-01	.259089+04	.123586-04
.100000+04	.200000+04	.104200+04	-.403071-01	.262108+04	.126312-04
.100000+04	.200000+04	.105400+04	-.512334-01	.265126+04	.126070-04
.100000+04	.200000+04	.106600+04	-.619136-01	.268145+04	.124548-04
.100000+04	.200000+04	.107800+04	-.723561-01	.271163+04	.122328-04
.100000+04	.200000+04	.111000+04	-.990990-01	.279213+04	.104682-04
.100000+04	.200000+04	.114200+04	-.124343+00	.287262+04	.951894-05
.100000+04	.200000+04	.117400+04	-.148211+00	.295311+04	.980999-05
.100000+04	.200000+04	.120600+04	-.170813+00	.303361+04	.903842-05
.100000+04	.200000+04	.123800+04	-.192245+00	.311410+04	.899922-05
.100000+04	.200000+04	.137000+04	-.270073+00	.344614+04	.815234-05

GIVE EXTERNAL LØAD AND CAGE SPEED

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Table 7. Cage Force. Applied Load 1200 lb_f. Cage Speed 1500 rpm

NC	P	EPNC	CSLIP	N	CAGE FØRCE
.150000+04	.120000+04	.150300+04	-.199601-02	.378069+04	.722412-06
.150000+04	.120000+04	.150600+04	-.398405-02	.378824+04	.134471-05
.150000+04	.120000+04	.150900+04	-.596420-02	.379578+04	.189737-05
.150000+04	.120000+04	.151200+04	-.793649-02	.380333+04	.238416-05
.150000+04	.120000+04	.151500+04	-.990096-02	.381088+04	.281346-05
.150000+04	.120000+04	.151800+04	-.118577-01	.381842+04	.319240-05
.150000+04	.120000+04	.152100+04	-.138067-01	.382597+04	.352720-05
.150000+04	.120000+04	.152400+04	-.157480-01	.383351+04	.382321-05
.150000+04	.120000+04	.152700+04	-.176817-01	.384106+04	.408509-05
.150000+04	.120000+04	.153000+04	-.196078-01	.384861+04	.431684-05
.150000+04	.120000+04	.153300+04	-.215264-01	.385615+04	.452199-05
.150000+04	.120000+04	.153600+04	-.234374-01	.386370+04	.470356-05
.150000+04	.120000+04	.153900+04	-.253411-01	.387125+04	.486426-05
.150000+04	.120000+04	.154200+04	-.272373-01	.387879+04	.500637-05
.150000+04	.120000+04	.154500+04	-.291261-01	.388634+04	.513200-05
.150000+04	.120000+04	.156300+04	-.403070-01	.393162+04	.562603-05
.150000+04	.120000+04	.158100+04	-.512333-01	.397689+04	.583776-05
.150000+04	.120000+04	.159900+04	-.619136-01	.402217+04	.588269-05
.150000+04	.120000+04	.161700+04	-.723561-01	.406745+04	.561492-05
.150000+04	.120000+04	.166500+04	-.990990-01	.418819+04	.459244-05
.150000+04	.120000+04	.171300+04	-.124343+00	.430893+04	.370010-05
.150000+04	.120000+04	.176100+04	-.148211+00	.442967+04	.443865-05
.150000+04	.120000+04	.180900+04	-.170812+00	.455041+04	.433923-05
.150000+04	.120000+04	.185700+04	-.192245+00	.467115+04	.425671-05
.150000+04	.120000+04	.205500+04	-.270073+00	.516921+04	.392336-05

GIVE EXTERNAL LØAD AND CAGE FØRCE

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Table 8. Cage Force. Applied Load 1600 lb_f. Cage Speed 2000 rpm

NC	P	EPNC	CSLIP	N	CAGE FØRCE
.200000+04	.160000+04	.200400+04	-.199601-02	.504092+04	.164172-05
.200000+04	.160000+04	.200800+04	-.398406-02	.505098+04	.294644-05
.200000+04	.160000+04	.201200+04	-.596422-02	.506104+04	.402553-05
.200000+04	.160000+04	.201600+04	-.793651-02	.507111+04	.491186-05
.200000+04	.160000+04	.202000+04	-.990099-02	.508117+04	.564341-05
.200000+04	.160000+04	.202400+04	-.118577-01	.509123+04	.624984-05
.200000+04	.160000+04	.202800+04	-.138067-01	.510129+04	.675449-05
.200000+04	.160000+04	.203200+04	-.157480-01	.511135+04	.717581-05
.200000+04	.160000+04	.203600+04	-.176817-01	.512141+04	.752852-05
.200000+04	.160000+04	.204000+04	-.196078-01	.513148+04	.782441-05
.200000+04	.160000+04	.204400+04	-.215264-01	.514154+04	.807298-05
.200000+04	.160000+04	.204800+04	-.234375-01	.515160+04	.828195-05
.200000+04	.160000+04	.205200+04	-.253411-01	.516166+04	.845766-05
.200000+04	.160000+04	.205600+04	-.272373-01	.517172+04	.860508-05
.200000+04	.160000+04	.206000+04	-.291262-01	.518178+04	.872871-05
.200000+04	.160000+04	.208400+04	-.403071-01	.524215+04	.913234-05
.200000+04	.160000+04	.210800+04	-.512334-01	.530252+04	.919300-05
.200000+04	.160000+04	.213200+04	-.619136-01	.536289+04	.878627-05
.200000+04	.160000+04	.215600+04	-.723561-01	.542326+04	.822623-05
.200000+04	.160000+04	.222000+04	-.990990-01	.558425+04	.695753-05
.200000+04	.160000+04	.228400+04	-.124343+00	.574524+04	.611173-05
.200000+04	.160000+04	.234800+04	-.148211+00	.590623+04	.684886-05
.200000+04	.160000+04	.241200+04	-.170813+00	.606721+04	.670175-05
.200000+04	.160000+04	.247600+04	-.192245+00	.622820+04	.655811-05
.200000+04	.160000+04	.274000+04	-.270073+00	.689227+04	.605820-05

GIVE EXTERNAL LØAD AND CAGE FØRCE

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Table 9. Cage Force. Applied Load 2000 lb_f. Cage Speed 4000 rpm

NC	P	EPNC	CSLIP	N	CAGE FØRCE
.400000+04	.200000+04	.400800+04	-.199601-02	.100818+05	.104885-05
.400000+04	.200000+04	.401600+04	-.398406-02	.101020+05	.208112-05
.400000+04	.200000+04	.402400+04	-.596422-02	.101221+05	.354437-05
.400000+04	.200000+04	.403200+04	-.793651-02	.101422+05	.869836-05
.400000+04	.200000+04	.404000+04	-.990099-02	.101623+05	.109293-04
.400000+04	.200000+04	.404800+04	-.118577-01	.101825+05	.117667-04
.400000+04	.200000+04	.405600+04	-.138067-01	.102026+05	.123759-04
.400000+04	.200000+04	.406400+04	-.157480-01	.102227+05	.128079-04
.400000+04	.200000+04	.407200+04	-.176817-01	.102428+05	.130755-04
.400000+04	.200000+04	.408000+04	-.196078-01	.102630+05	.132401-04
.400000+04	.200000+04	.408800+04	-.215264-01	.102831+05	.133340-04
.400000+04	.200000+04	.409600+04	-.234375-01	.103032+05	.133800-04
.400000+04	.200000+04	.410400+04	-.253411-01	.103233+05	.133937-04
.400000+04	.200000+04	.411200+04	-.272373-01	.103434+05	.133847-04
.400000+04	.200000+04	.412000+04	-.291262-01	.103636+05	.133594-04
.400000+04	.200000+04	.416800+04	-.403071-01	.104843+05	.128963-04
.400000+04	.200000+04	.421600+04	-.512334-01	.106050+05	.123608-04
.400000+04	.200000+04	.426400+04	-.619136-01	.107258+05	.118038-04
.400000+04	.200000+04	.431200+04	-.723561-01	.108465+05	.113244-04
.400000+04	.200000+04	.444000+04	-.990990-01	.111685+05	.974938-05
.400000+04	.200000+04	.456800+04	-.124343+00	.114905+05	.100120-04
.400000+04	.200000+04	.469600+04	-.148211+00	.118125+05	.980264-05
.400000+04	.200000+04	.482400+04	-.170813+00	.121344+05	.959690-05
.400000+04	.200000+04	.495200+04	-.192245+00	.124564+05	.939370-05
.400000+04	.200000+04	.548000+04	-.270073+00	.137845+05	.871072-05

GIVE EXTERNAL LØAD AND CAGE SPEED

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Table 10. Cage Force. Applied Load 2400 lb_f. Cage Speed 6000 rpm

NC	P	EPNC	CSLIP	N	CAGE FORCE
.600000+04	.240000+04	.601200+04	-.199601-02	.151228+05	.337390-05
.600000+04	.240000+04	.602400+04	-.398405-02	.151529+05	.526740-05
.600000+04	.240000+04	.603600+04	-.596420-02	.151831+05	.105239-04
.600000+04	.240000+04	.604800+04	-.793649-02	.152133+05	.161212-04
.600000+04	.240000+04	.606000+04	-.990096-02	.152435+05	.173153-04
.600000+04	.240000+04	.607200+04	-.118577-01	.152737+05	.177723-04
.600000+04	.240000+04	.608400+04	-.138067-01	.153039+05	.180114-04
.600000+04	.240000+04	.609600+04	-.157480-01	.153341+05	.181408-04
.600000+04	.240000+04	.610800+04	-.176817-01	.153642+05	.181983-04
.600000+04	.240000+04	.612000+04	-.196078-01	.153944+05	.182310-04
.600000+04	.240000+04	.613200+04	-.215264-01	.154246+05	.182260-04
.600000+04	.240000+04	.614400+04	-.234374-01	.154548+05	.176609-04
.600000+04	.240000+04	.615600+04	-.253411-01	.154850+05	.176207-04
.600000+04	.240000+04	.616800+04	-.272373-01	.155152+05	.175710-04
.600000+04	.240000+04	.618000+04	-.291261-01	.155454+05	.175136-04
.600000+04	.240000+04	.625200+04	-.403070-01	.157265+05	.170629-04
.600000+04	.240000+04	.632400+04	-.512333-01	.159076+05	.157198-04
.600000+04	.240000+04	.639600+04	-.619136-01	.160887+05	.152304-04
.600000+04	.240000+04	.646800+04	-.723561-01	.162698+05	.139257-04
.600000+04	.240000+04	.666000+04	-.990990-01	.167528+05	.102462-04
.600000+04	.240000+04	.685200+04	-.124343+00	.172357+05	.107423-04
.600000+04	.240000+04	.704400+04	-.148211+00	.177187+05	.102564-04
.600000+04	.240000+04	.723600+04	-.170812+00	.182016+05	.987170-05
.600000+04	.240000+04	.742800+04	-.192245+00	.186846+05	.959282-05
.600000+04	.240000+04	.822000+04	-.270073+00	.206768+05	.130024-04

GIVE EXTERNAL LOAD AND CAGE SPEED

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Table 11. Typical Bearing Operating Parameters at Maximum Driving Cage Force. Cage Speed 4000 rpm

1600.,4000.,10364.

P = .16000+04 N = .10364+05 EPNJ = .20108+05 CSLIP =-.29167+01

PNJC = .19521+05 EPNC = .41202+04 TTC = .30988-01 CF = .93599-05

J	DEF	W	TC	NJ	SLIP	RCF	H
1	.1749-03	.7389-04	.2223-01	.1986+05	-.1232+01	-.3150-05	.9384-03
2	.1492-03	.6195-04	.2003-01	.1985+05	-.1296+01	-.2327-05	.9823-03
3	.7919-04	.3066-04	.1251-01	.1988+05	-.1115+01	-.7705-06	.1180-02
4	.0000	.0000	.0000	.1982+05	-.1442+01	-.4931-07	.0000
5	.0000	.0000	.0000	.1975+05	-.1784+01	-.3933-07	.0000
6	.0000	.0000	.0000	.1970+05	-.2050+01	-.3098-07	.0000
7	.0000	.0000	.0000	.1965+05	-.2259+01	-.2405-07	.0000
8	.0000	.0000	.0000	.1962+05	-.2422+01	-.1844-07	.0000
9	.0000	.0000	.0000	.1960+05	-.2546+01	-.1401-07	.0000
10	.0000	.0000	.0000	.1958+05	-.2640+01	-.1056-07	.0000
11	.7919-04	.3066-04	.1343-01	.1960+05	-.2507+01	-.5298-06	.1177-02
12	.1492-03	.6195-04	.2007-01	.1990+05	-.1048+01	-.2397-05	.9827-03
13	.3783-03	.7389-04	.2223-01	.1986+05	-.1233+01	.4102-07	.9384-03

GIVE EXTERNAL LØAD,CAGE SPEED AND INNER RING SPEED

800.,4000.,10515.

P = .80000+03 N = .10515+05 EPNJ = .20400+05 CSLIP =-.43108+01

PNJC = .19521+05 EPNC = .41802+04 TTC = .21580-01 CF = .32562-05

J	DEF	W	TC	NJ	SLIP	RCF	H
1	.9569-04	.3783-04	.1509-01	.1989+05	-.2482+01	-.1095-05	.1123-02
2	.8055-04	.3125-04	.1382-01	.1989+05	-.2521+01	-.8306-06	.1180-02
3	.3906-04	.1399-04	.9863-02	.1987+05	-.2592+01	-.3024-06	.1454-02
4	.0000	.0000	.0000	.1982+05	-.2864+01	-.4912-07	.0000
5	.0000	.0000	.0000	.1975+05	-.3204+01	-.3903-07	.0000
6	.0000	.0000	.0000	.1969+05	-.3475+01	-.3036-07	.0000
7	.0000	.0000	.0000	.1965+05	-.3686+01	-.2325-07	.0000
8	.0000	.0000	.0000	.1962+05	-.3848+01	-.1755-07	.0000
9	.0000	.0000	.0000	.1959+05	-.3970+01	-.1311-07	.0000
10	.0000	.0000	.0000	.1957+05	-.4061+01	-.9707-08	.0000
11	.3906-04	.1399-04	.1030-01	.1957+05	-.4091+01	-.1707-06	.1450-02
12	.8055-04	.3125-04	.1379-01	.1971+05	-.3401+01	-.6752-06	.1178-02
13	.2216-03	.3783-04	.1509-01	.1989+05	-.2480+01	-.1764-06	.1123-02

GIVE EXTERNAL LØAD,CAGE SPEED AND INNER RING SPEED

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Table 12. Typical Bearing Operating Parameters at Maximum Driving Cage Force. Cage Speed 2000 rpm

1600.,2000.,5292.

P = .16000+04 N = .52920+04 EPNJ = .10267+05 CSLIP =-.49346+01

PNJC = .97605+04 EPNC = .21038+04 TTC = .29854-01 CF = .89840-05

J	DEF	W	TC	NJ	SLIP	RCF	H
1	.1721-03	.7255-04	.2218-01	.1040+05	.1322+01	-.2931-05	.5896-03
2	.1484-03	.6155-04	.1668-01	.1063+05	.3489+01	-.2041-05	.6177-03
3	.8392-04	.3270-04	.1250-01	.1041+05	.1374+01	-.8542-06	.7253-03
4	.0000	.0000	.0000	.1025+05	-.1447+00	-.1780-07	.0000
5	.0000	.0000	.0000	.1020+05	-.6378+00	-.1654-07	.0000
6	.0000	.0000	.0000	.1016+05	-.1082+01	-.1531-07	.0000
7	.0000	.0000	.0000	.1011+05	-.1492+01	-.1409-07	.0000
8	.0000	.0000	.0000	.1008+05	-.1869+01	-.1289-07	.0000
9	.0000	.0000	.0000	.1004+05	-.2214+01	-.1174-07	.0000
10	.0000	.0000	.0000	.1001+05	-.2527+01	-.1063-07	.0000
11	.8392-04	.3270-04	.1411-01	.1006+05	-.2059+01	-.8000-06	.7209-03
12	.1484-03	.6155-04	.1965-01	.1022+05	-.4580+00	-.2259-05	.6134-03
13	.3513-03	.7255-04	.2220-01	.1041+05	.1403+01	.4884-06	.5896-03

GIVE EXTERNAL LØAD,CAGE SPEED AND INNER RING SPEED

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800.,2000.,5353.

P = .80000+03 N = .53530+04 EPNJ = .10386+05 CSLIP =-.60180+01

PNJC = .97605+04 EPNC = .21281+04 TTC = .21348-01 CF = .32775-05

J	DEF	W	TC	NJ	SLIP	RCF	H
1	.9266-04	.3650-04	.1472-01	.1021+05	-.1680+01	-.1014-05	.7054-03
2	.7963-04	.3085-04	.1373-01	.1023+05	-.1505+01	-.8045-06	.7372-03
3	.4409-04	.1601-04	.1057-01	.1023+05	-.1494+01	-.3327-06	.8743-03
4	.0000	.0000	.0000	.1019+05	-.1851+01	-.1632-07	.0000
5	.0000	.0000	.0000	.1015+05	-.2298+01	-.1505-07	.0000
6	.0000	.0000	.0000	.1010+05	-.2702+01	-.1382-07	.0000
7	.0000	.0000	.0000	.1007+05	-.3072+01	-.1261-07	.0000
8	.0000	.0000	.0000	.1003+05	-.3410+01	-.1145-07	.0000
9	.0000	.0000	.0000	.1000+05	-.3717+01	-.1033-07	.0000
10	.0000	.0000	.0000	.9971+04	-.3993+01	-.9279-08	.0000
11	.4409-04	.1601-04	.1086-01	.9969+04	-.4006+01	-.2718-06	.8704-03
12	.7963-04	.3085-04	.1362-01	.1013+05	-.2486+01	-.7657-06	.7359-03
13	.1930-03	.3650-04	.1472-01	.1021+05	-.1688+01	.1044-06	.7054-03

GIVE EXTERNAL LØAD,CAGE SPEED AND INNER RING SPEED

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Table 13. Typical Bearing Operating Parameters at Maximum Driving Cage Force. Cage Speeds 6000 and 400 rpm

1600.,6000.,15485.

P = .16000+04 N = .15485+05 EPNJ = .30043+05 CSLIP =-.25342+01

PNJC = .29282+05 EPNC = .61560+04 TTC = .31203-01 CF = .93235-05

J	DEF	W	TC	NJ	SLIP	RCF	H
1	.1796-03	.7608-04	.2241-01	.2940+05	-.2125+01	-.2919-05	.1232-02
2	.1506-03	.6260-04	.2019-01	.2959+05	-.1507+01	-.2758-05	.1298-02
3	.7141-04	.2734-04	.1231-01	.2934+05	-.2327+01	-.4628-06	.1608-02
4	.0000	.0000	.0000	.2951+05	-.1777+01	-.1118-06	.0000
5	.0000	.0000	.0000	.2940+05	-.2145+01	-.6118-07	.0000
6	.0000	.0000	.0000	.2934+05	-.2345+01	-.3082-07	.0000
7	.0000	.0000	.0000	.2931+05	-.2443+01	-.1518-07	.0000
8	.0000	.0000	.0000	.2929+05	-.2491+01	-.7152-08	.0000
9	.0000	.0000	.0000	.2929+05	-.2514+01	-.3326-08	.0000
10	.0000	.0000	.0000	.2928+05	-.2525+01	-.1534-08	.0000
11	.7141-04	.2734-04	.1241-01	.2929+05	-.2491+01	-.3666-06	.1607-02
12	.1506-03	.6260-04	.2015-01	.2953+05	-.1715+01	-.2586-05	.1297-02
13	.4224-03	.7608-04	.2241-01	.2940+05	-.2128+01	-.5762-06	.1232-02

GIVE EXTERNAL LØAD,CAGE SPEED AND INNER RING SPEED

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200.,400.,1125.

P = .20000+03 N = .11250+04 EPNJ = .21826+04 CSLIP =-.10563+02

PNJC = .19521+04 EPNC = .44724+03 TTC = .14101-01 CF = .54989-06

J	DEF	W	TC	NJ	SLIP	RCF	H
1	.2632-04	.9027-05	.9364-02	.2153+04	-.1356+01	-.1608-06	.1404-02
2	.2277-04	.7685-05	.9019-02	.2155+04	-.1268+01	-.1320-06	.1467-02
3	.1306-04	.4147-05	.7857-02	.2154+04	-.1310+01	-.6237-07	.1735-02
4	.0000	.0000	.0000	.2147+04	-.1635+01	-.6420-09	.0000
5	.0000	.0000	.0000	.2138+04	-.2055+01	-.6264-09	.0000
6	.0000	.0000	.0000	.2129+04	-.2459+01	-.6105-09	.0000
7	.0000	.0000	.0000	.2120+04	-.2851+01	-.5939-09	.0000
8	.0000	.0000	.0000	.2112+04	-.3233+01	-.5768-09	.0000
9	.0000	.0000	.0000	.2104+04	-.3604+01	-.5592-09	.0000
10	.0000	.0000	.0000	.2096+04	-.3963+01	-.5412-09	.0000
11	.1306-04	.4147-05	.7979-02	.2102+04	-.3690+01	-.5949-07	.1735-02
12	.2277-04	.7685-05	.9033-02	.2143+04	-.1834+01	-.1311-06	.1467-02
13	.5301-04	.9027-05	.9364-02	.2154+04	-.1322+01	-.6905-07	.1404-02

GIVE EXTERNAL LØAD,CAGE SPEED AND INNER RING SPEED

APPENDIX C

THERMAL REDUCTION FACTOR

In his study [6] Cheng used a model to express the viscosity as a function of pressure and temperature of the form

$$\eta = \eta_o \, \mathrm{Exp}[\alpha p + (\beta + \gamma p)(\frac{1}{T} - \frac{1}{To})] \qquad \text{(A-1)}$$

Taking data for XRM-109 at different pressures and temperatures [23] the values of η_o, α, β and γ are:

$$\alpha = 8.52 \times 10^{-5} \text{ psi}^{-1}$$

$$\eta_o = 909.15 \text{ Cp} = 1.3185 \times 10^{-4} \frac{lb_f - Sec}{in^2}$$

$$\beta = 7.886 \times 10^{-3} \; {}^oR$$

$$\gamma = 8.7 \times 10^{-2} \; {}^oR \text{ psi}^{-1}$$

The dimensionless values of α, β and γ for the reference values of P = 110,000 psi and To = 560°F, are:

$$\alpha' = \alpha \pi Po/2 = 14.72$$

$$\beta' = 14.08$$

$$\gamma' = 26.84$$

As shown in Figure 6 of reference [6] the slip ratio has a very small influence on the value of ϕ_t, for this reason its influence was reglected and ϕ_t evaluated at $(U_2 - U_1)/U_2 = 0.2$.

With the found parameters for the XRM-109 oil, the nearest set of curves of Ref. [6] was Figure 10, run 27, [6]. This curve was corrected to take into account a slip of .2, the correction factor taken from Figure 6 [6]. (The corrected curve is equal to that of run 51, Figure 14 [6].) The value of β' for XRM-109 is significatively different from β' = 8.006 of run 51, so this curve was corrected, the correction factors being taken from Figure 5, [6]. The corrected curve was then correlated by an equation of the form

$$\phi_t = e^{AQm^B} \tag{A-2}$$

A and B being the functions of X, as shown in Figure 30. The final values of A and B are:

$$A = -\,0.5677 - 0.0348X \tag{A-3}$$

$$B = 0.4003 + 0.0311\,X \tag{A-4}$$

Figure 31 shows ϕ_t as a function of Q_m for three pressures: 100,000, 150,000, and 200,000 psi.

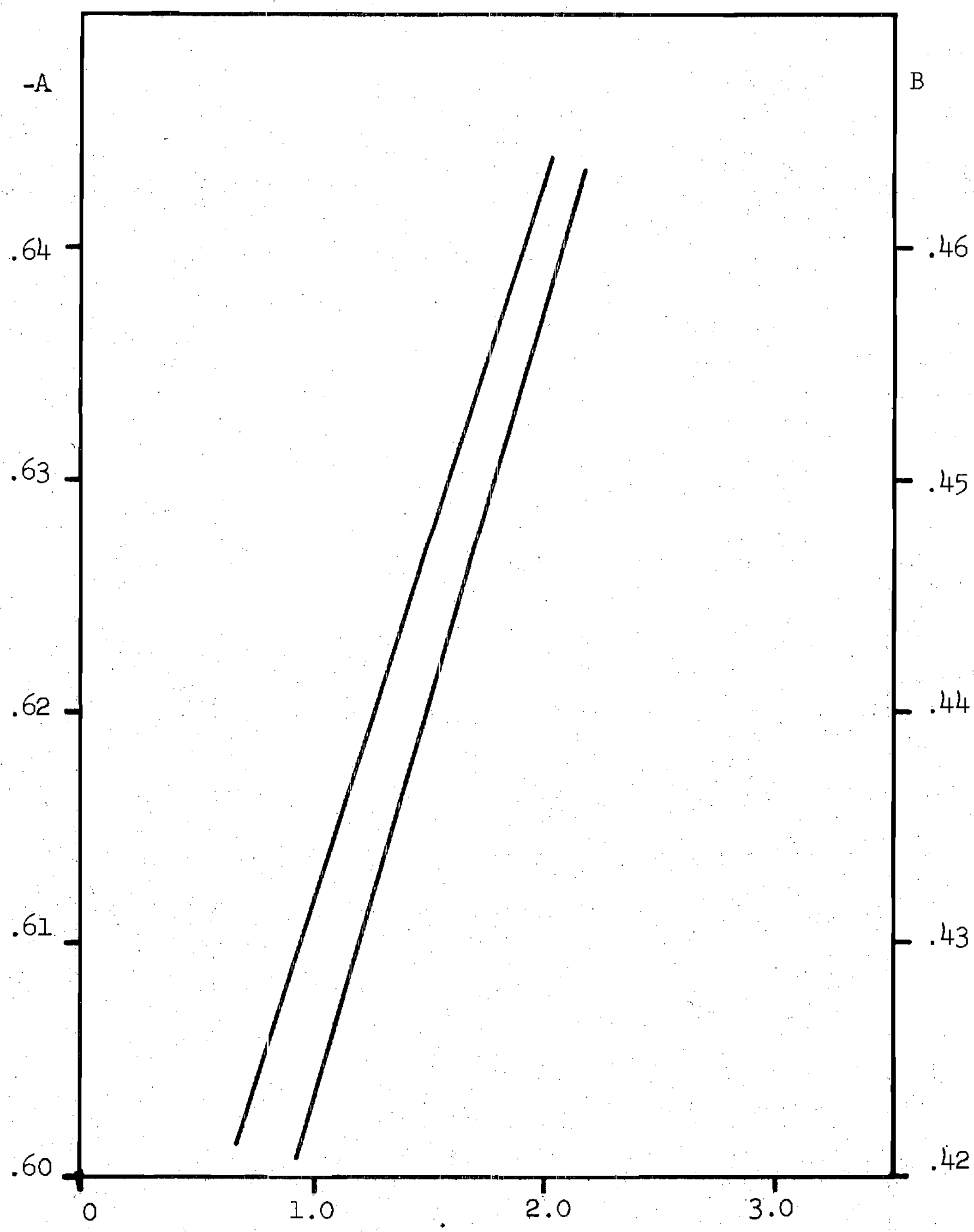

Figure 30. Values of Factors A and B as Functions of X

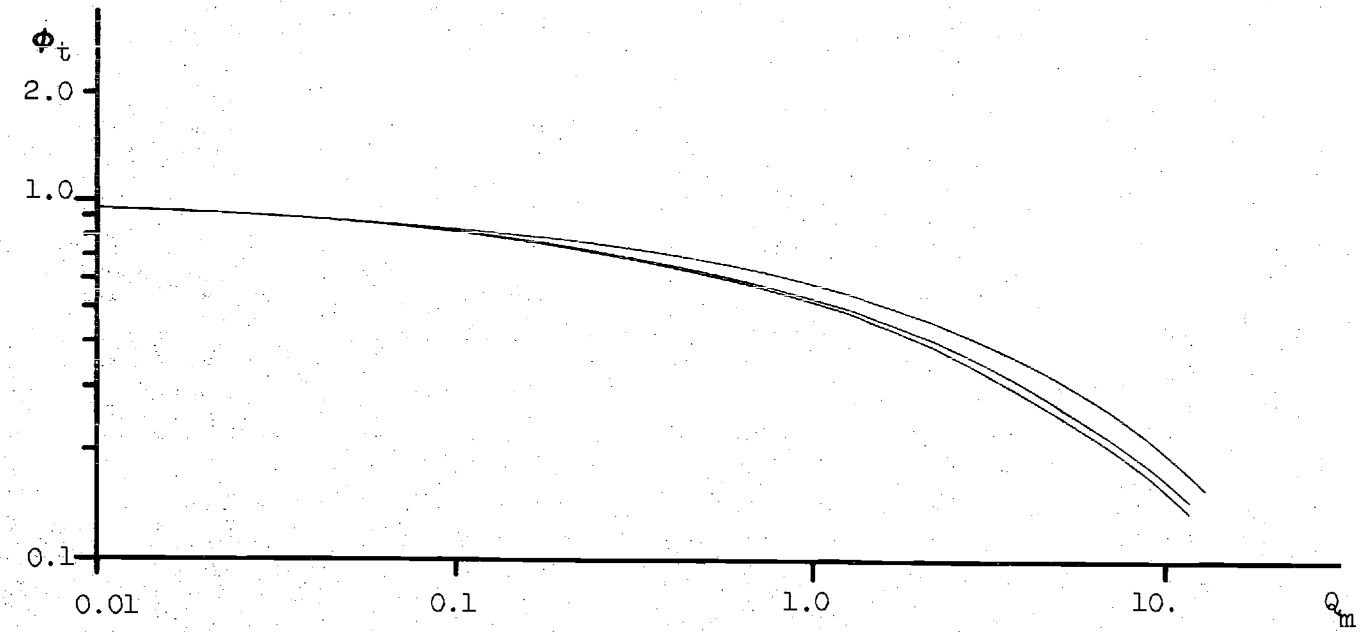

Figure 31. Film Thickness Thermal Reduction Factor Versus Q_m

APPENDIX D

TRACTION COEFFICIENT AT HIGH PRESSURES

Table 14 gives some values of the traction coefficients for fluid XRM-109 taken from reference [11]. Those values were used to find an empirical formula to calculate the traction coefficient.

An examination of the curves given in [11] indicated that the correlation may be made with a formula of the form

$$TC = -C\lambda + D(1 - e^{-A\lambda}) \tag{A-5}$$

This is the sum of a straight line with negative slope and a decaying exponential. This combination gives for $\lambda = 0$, $TC = 0$, gives a maximum TC and then a continuous decreasing of TC as λ increases.

<u>The Variable λ</u>

In the formula A-5:

$$\lambda = v/ua \tag{A-6}$$

$$v = \text{sliding velocity} = (u_1 - u_2) \text{ in/sec} \tag{A-7}$$

u_a = Mean entraining velocity of the fluid into the contact.

In Figure 32 can be seen that the mean velocities at which the fluid is entering the contacts are:

$$u_{ai} = \tfrac{1}{2}[(u_{1i} - u_{ci}) + (u_{2i} - u_{ci})] = \tfrac{1}{2}(u_{2i} + u_{1i} - 2u_{ci}) \tag{A-8}$$

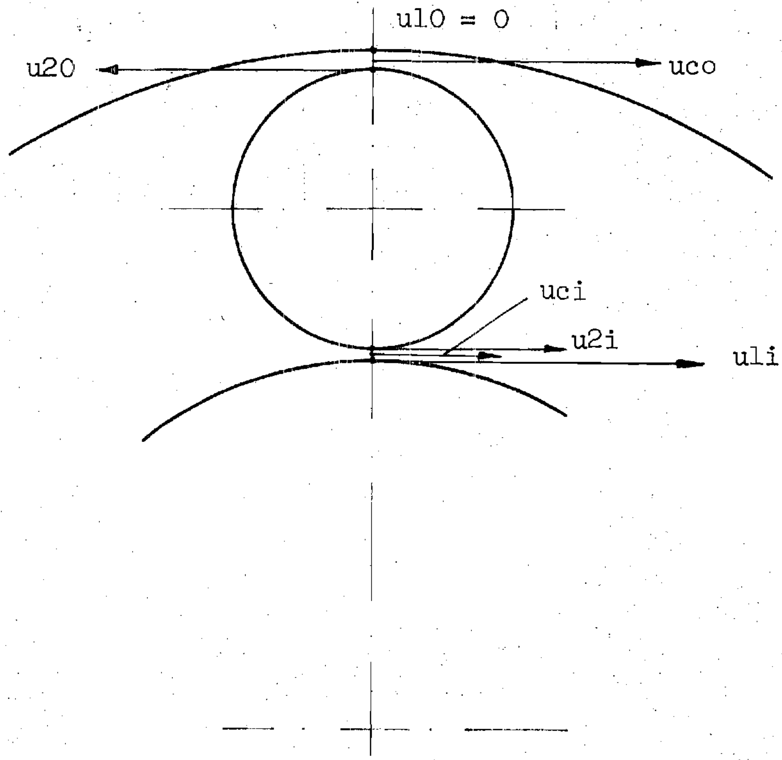

Figure 32. Entraining Velocity at Contacts

$$uao = \tfrac{1}{2}[u20 + uco + 0 + uco] = u20/2 + uco \qquad \text{(A-9)}$$

Uc is the velocity at the point of contact,

$$uco_i = \frac{2\pi NcRp}{60}(1 \pm S) \qquad \text{(A-10)}$$

The Parameters A, C, D

These parameters which are functions of X, were found from the data, in such a way that three conditions were fulfilled:

a) That the maximum traction coefficient was correctly predicted for each pressure.

b) That the traction coefficient was correctly predicted at $\lambda = .12$, and

c) That at the maximum traction coefficient TC*, at $\lambda = \lambda^*$, the slope of the curve TC - λ was zero, i.e.,

$$\frac{dTC}{d\lambda}\Big|_{\lambda=\lambda^*} = 0 = - C + DAe^{-A\lambda^*} \tag{A-11}$$

$$TC^* = - C\lambda^* + D(1 - e^{-A\lambda^*}) \tag{A-12}$$

From (A-11) and (A-12) known C, A may be calculated from

$$\frac{1}{A}\,(e^{A\lambda^*} - 1) = \frac{TC^* + C\lambda^*}{C} \tag{A-13}$$

and D from

$$D = \frac{TC^* + C\lambda^*}{1 - e^{-A\lambda^*}} \tag{A-14}$$

A study of the variation of C demonstrated that C may be approximated by

$$C = 0.0143X + 0.03571 \tag{A-15}$$

And the values of D and A are:

$$D = 0.046 - 0.031\, e^{-0.31136X} \tag{A-16}$$

$$A = 147.507 \text{ arctg}(4.76979(X - 1.0597)) + 0.1\,(X - 1.0597) + 210 \tag{A-17}$$

Figures 33 and 34, show D and A as functions of the pressure ratio X.

The sign of the traction force was determined by the sign of the sliding velocity while ua was taken in its absolute value.

If sliding ratios greater than 0.14 were found, the calculation of the traction coefficient is made taking the value at $\lambda = .14$, Tc_{14}, as the basis for an exponentially decaying curve as given by Equation A-18.

$$TC = TC_{14}\ EXP[\ln 3(.14 - \lambda)/2] \qquad \text{(A-18)}$$

This equation was selected so that it follows the trend of the experimental curves and it dacays asymtotally to zero. This was done because of the lack of data of traction coefficients at higher slip ratios. The asymtotic values of zero was chosen because there was not any better value found. Regirer [21] gave a method to estimate the asympototic value of the traction force but unfortunately it is not applicable to the present case.

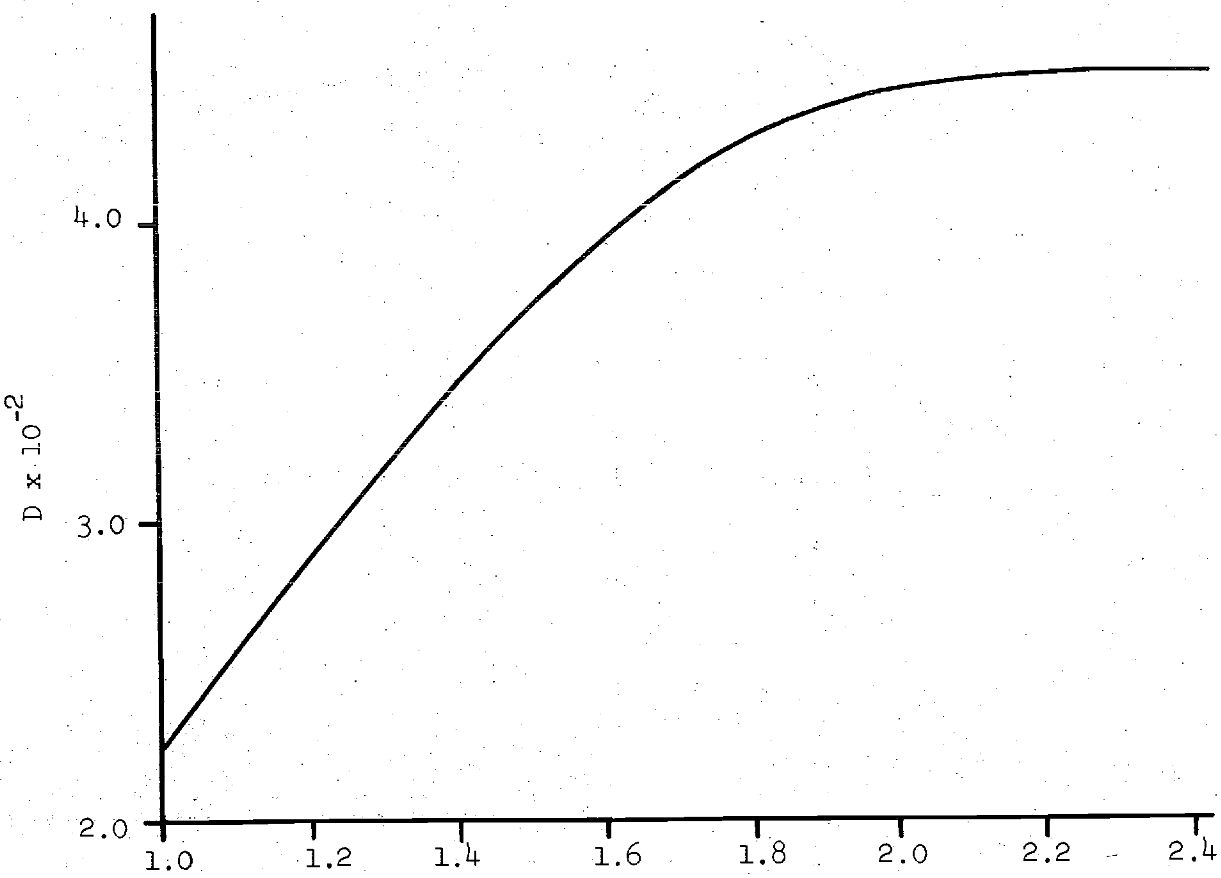

Figure 33. Factor D Versus Pressure Ratio

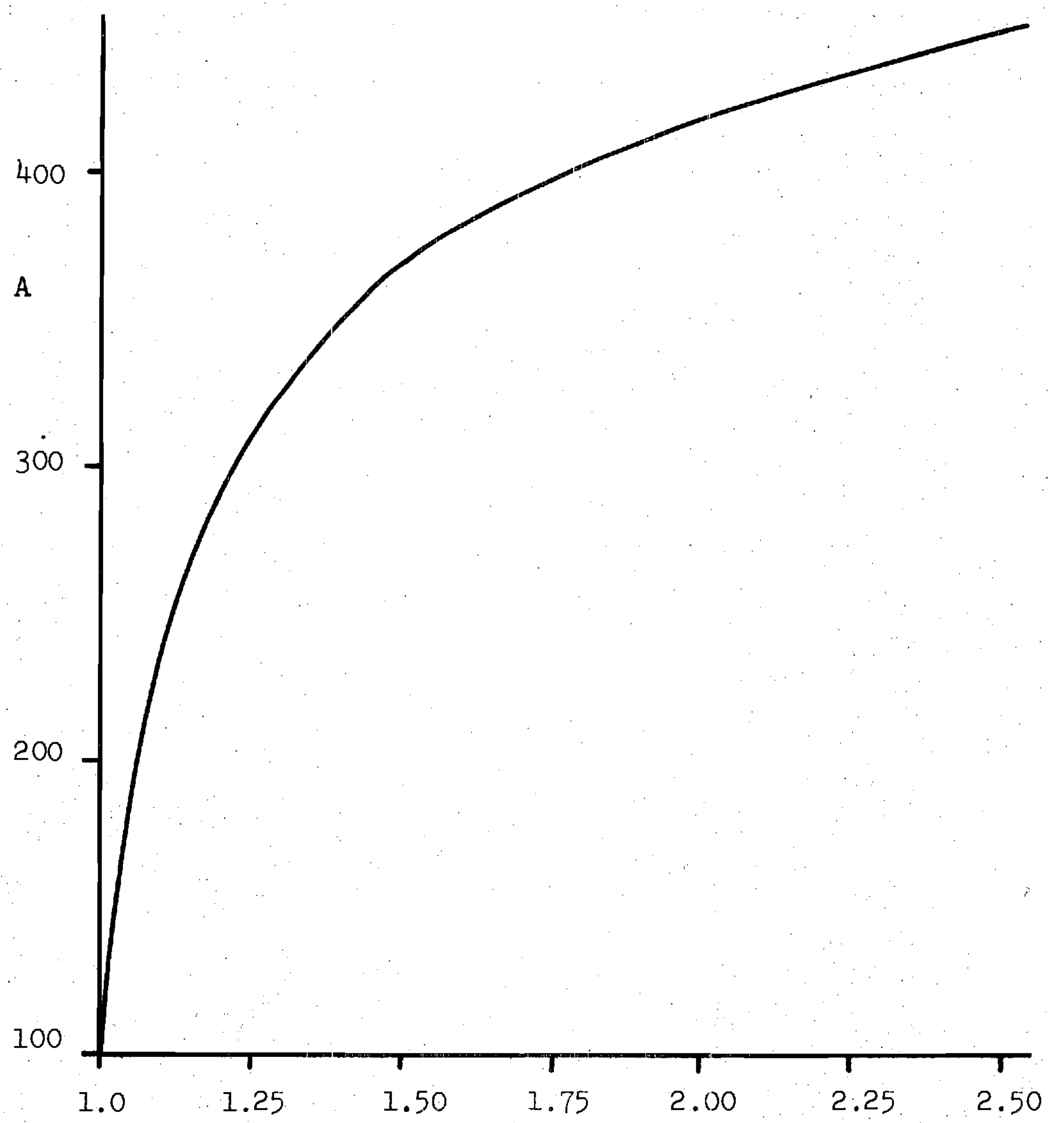

Figure 34. Factor A Versus Pressure Ratio

Table 14. Traction Coefficients for Fluid XRM-109 A - 4

v	λ	v_a = 500. in/Sec.				λ	v_a = 1000 in/Sec.		
in/Sec		p_o 115000	154000	200000	250000		p_o 115000	154000	200000
2.5	.005	.0180	.2090	.0370	.0380	.0010	.0180	.0250	.0300
5.	.010	.0210	.0330	.0420	.0430	.0050	.0205	.0280	.0340
7.5	.015	.0220	.0340	.0425	.0440	.0075	.0210	.0290	.0360
10.	.020	.0225	.0335	.0420	.0435	.0100	.0210	.0295	.0375
15.	.030	.0222	.0330	.0395	.0420	.0150	.0210	.0285	.0375
20.	.040	.0220	.0320	.0370	.0410	.0200	.0210	.0280	.0370
30.	.060	.0210	.0300	.0350	.0380	.0300	.0190	.0270	.0340
40.	.080	.0200	.0280	.0340	.0370	.0400	.0180	.0260	.0330
50.	.100	.0190	.0270	.0324	.0360	.0500	.0171	.0245	.0310
60.	.120	.0180	.0260	.0310	.0350	.0600	.0170	.0230	.0295

BIBLIOGRAPHY

1. Harris, T. A., "An Analytical Method to Predict Skidding in Thrust - loaded Angular - Contact Ball Vearings," ASME Trans., Vol. 93, No. 1, 17-24, Jan. 1971.

2. Bonnes, R. J., "The Effect of Oil Supply on Cage and Roller Motion in a Lubircated Roller Bearing," ASME Trans., Vol. 92, No. 1, 39-53, Jan. 1970.

3. Poplawski, J. V., "Slip and Cage Forces in a High-Speed Roller Bearing," J. Lubr. Tech., 143-54, April 1972.

4. Rumbarger, J. H., Filletti, E. G., Gubernick, D., "Gas Turbine Engine Mainshaft Roller Bearing System Analysis," ASME Trans., Vol. 95, No. 4, October 1973.

5. Dawson, G. and Higginson, G. R., "Elasto-Hydrogynamic Lubrication - The Fundamentals of Roller and Gear Lubrication", Pergamon Press, London, 1966.

6. Cheng, H. S., "Calculation of Elastohydrodynamic Film Thickness in High Speed Rolling and Sliding Contacts," Mechanical Technology Incorporated 67TR24.

7. Harris, T. A., "Rolling Bearing Analysis," John Wiley & Sons, New York, 1966.

8. Perry-Rand Co. "UP7542 Large Systems MATHPAC, Programmers Reference."

9. Lieberstein, H. M., "A Course in Numerical Analysis," J. Wiley and Sons, New York, 1966.

10. Berezin, I.S., Zhidkov, N., "Computing Methods," Vol. II, Pergamon Press, London, 1965.

11. Trachman, E. G. and Cheng, H. S., "Traction in Elastohydrodynamic Line Contacts for Two Synthesized Hydrocarbon Fluids," Unpublished paper.

12. Cheng, H. S., "Isothermal Elastohydrodynamic Theory for the Full Range of Pressure - Viscosity Coefficient," ASME Trans., Vol. 94, No. 1, 35-43, Jan. 1972.

13. Kannel, J. W., and Walowet, J. A., "Simplified Analysis for Traction Between Rolling - Sliding Elastohydrodynamic Contacts," ASME Trans., Vol. 93, No. 1, Jan. 1971.

14. Baglin, K. P. and Archard, J. F., "Non-Dimensional Presentation of Friction Traction in Lubricated Line Contact," (Part 1), Report 73-10, University of Leicester, June 1973.

15. Adams, R. and Hirst, W., "Frictional Traction in Elastohydrodynamic Lubrication," Proc. of Royal Soc., London, A332, 505-525, 1973.

16. Kannel, W. and Bupara, Ss., "Rheology of Lubricants in Real Bearing Contacts," Botelle's Columbus Laboratories, Feb. 1974. Unpublished paper.

17. Sanborn, D. M. and Winer, W. O., "Fluid Rheological Effects in Sliding Elastohydrodynamic Point Contacts with Transient Loading: 2-Traction," ASME Trans., Vol. 93, No. 3, 342-348, July 1971.

18. Trachman, G. and Cheng, H. S., "Thermal and Non-Newtonian Effects on Traction in Elastohydrodynamic Contacts," Paper presented to the Tribology Group Conference, 1972.

19. Jacobson, J., "Lubrication Rheology at High Shear Stresses," Ph.D. Thesis, Georgia Institute of Technology, Sept. 1973.

20. Rodkiewicz, C. M. and Srinivassan, "Elastohydrodynamic Lubrication in Rolling and Sliding Contacts," ASME Trans., Vol. 94, No. 4, Oct. 1972.

21. Regirer, S. A., "The Influence of Thermal Effects on the Viscous Resistance of a Steady Uniform Flow of a Liquid," PMM, Vol. 22, No. 3, 414-418, 1958.

22. Parker, R. J., Zartsky, E. V. and Anderson, W. J., "Spinning Friction Coefficient with Three Lubricants," ASME Trans., Vol. 90, No. 1, Jan. 1968.

23. Bohn, M. Carlson, S. Lee, D., Jacobsen, J., Sanborn, D. M. and Winer, W. O., "Investigation of Lubrication Rheology as Applied to Elastohydrodynamic Lubrication," Report to NASA, June 1972.

24. Greenwood, T. A. and Kauzlarich, J., "Inlet Shear Heating in Elastohydrodynamic Lubrication," ASME Trans., Vol. 95, No. 4, 417-433, October 1973.

25. Sanborn, D. M. and Winer, W. O., "Fluid Rheological Effects in Sliding Elastohydrodynamic Point Contacts with Transient Loading, 1 - Film Thickness," ASME Trans., Vol. 93, No. 2, 262-271, April 1971.

26. Kelly, L. G., "Handbook of Numerical Methods and Applications," Addison-Wesley Publ., New York, 1967.

27. Townsed, D. P., Allen, C. W. and Zaretsky, E. V., "Study of Ball Bearing Torque Under Elastohydrodynamic Lubrication," Paper presented to the ASLE - ASME Joint Conference, Atlanta, Oct. 16-18, 1973.